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Scientific Enlightenment, Div. Two
2. A. 4. The Problem of Representation
Chapter 2
The application of Calculus such as in interior ballistics
ACADEMY | previous section | Table of Content | next section | **GALLERY**

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The representation of motion by calculus is of course only one side of calculus, the side of differentiation, or y/x (distance divided by time). The other side, integration, y*x, usually involves, on the Cartesian plan, the calculation of the area under the curve: the addition of the infinitesimals. It is all the same: the power of calculus in representation derives from its analysis of the given phenomenon into components and subsequent re-integration of them. While velocity is distance travelled divided by time elapsed pressed into the instantaneous moment of time t, and establishes a relation of division between the two components of motion that are along space and time, work is force x distance (or "total" force)...

Let us look at a real example from interior ballistic (study of what happens inside the barrel of a gun, as opposed to exterior ballistics that studies the shell after it leaves the muzzle). The phenomenon to be represented by the one dimensional, quantitative representation called mathematical equation is: the motion of the bullet by Le Duc. This is the "whole". (The following is a commentary on Alexander Hahn's *Basic Calculus*, p. 488-90.)

To describe (represent) this phenomenon (whole) quantitatively, we:

(1) cut it open along space and time: distance travelled and time elapsed. If we represent this on the Cartesian coordinate, we will have reduced the four dimensional phenomenon to the two dimensions of Cartesian coordinate.

(2) determine the actual quantity of the constituents, distance and time, by measuring, so as to derive the velocity of the shell. This is, again, possible thanks to numeralization: the cutting of space and time into definite (numbered) intervals, which is the legacy of the Neolithic Revolution.

a point on the coordinate of time = time(t)
a point on the coordinate of distance = position (x)

$$v(t) = \lim_{\Delta t \to 0} \frac{p(t + \Delta t) - p(t)}{\Delta t} \text{ with } p(t) = x$$

(3) Now x as position corresponds to a definite t, so that $v(t)$ corresponds to $v(x)$, and according to Le Duc:

$$v(x) = \frac{rx}{s + x}$$

where r and s = constants determined by the properties of the barrel, shell, explosive force, etc.

Now a definite relationship is established between velocity (itself a relationship between distance and time, the primordial constituents into which the phenomenon as a whole has been analyzed) and position of the shell (a ordinal point on the continuum of space or distance travelled thus far): velocity as function of position.

This equation is the one dimensional version of the quantitative representation of the "whole" (the motion of the bullet). its two dimensional version will be the Cartesian graph of the equation.

(4) Now this equation captures the "structure" of the "whole" (phenomenon), but only under one aspect, the aspect of velocity (a composite of space and time, the rate at which the quantity of space [distance travelled] increases with respect to the corresponding increase [elapse] of time) with respect to the position of the shell (distance travelled). Other aspects of the structure can be deduced from this aspect of velocity through algebraic transformation:

since x = x(t) (position = position at a certain time t), $v(t) = v(x[t])$, and since $v(t) = x'(t)$,

$$x'(t) = v(x(t)) = \frac{rx(t)}{s + x(t)}$$

and since a(t) = x''(t), another aspect of the structure is:

$$a(t) = x''(t) = \frac{r^2 sx(t)}{(s + x(t))^3}$$

which establishes the definite relationship between acceleration of the bullet and the position of the bullet in relation to time elapsed, which is another aspect of the structure, the aspect of acceleration (the rate at which the rate of the increase of space [distance travelled] with respect to time elapsed increases).

(5) What is the cause of the motion of the bullet? Its context is part of the cause (i.e. the properties of the barrel and the shell itself, captured by r and

s). Force is the other part. Force is what propels the bullet to move in the first place -- the movement being modified by the characteristics of the barrel and the shell. Force produces acceleration, which accumulates into velocity, with which the bullet runs out a "course" (the whole phenomenon of the motion of the bullet). In this case (the case of the ballistic) force is produced by the explosion of the powder (minus friction, which is negligible here). Now how to represent quantitatively this cause of motion which is force? How to capture its structure?

By the product of mass and acceleration: F=ma, or F(t) = ma(t). While the "cause" of motion is the explosion of powder, the *quantity* of this cause (of force), of the force's ability to do "work" (to cause motion), is inherent (implied) in the phenomenon itself. (The mass of the bullet is included in the phenomenon: the "whole" means: motion of the bullet of mass m.) It is gotten by m * a, or:

$$F(t) = mx''(t) = \frac{mr^2sx(t)}{(s + x(t))^3}$$

This is (quantity) of force at time t after explosion.

Force at position x would be (as a function of its position x in the barrel, the force on the shell is given by):

$$f(x) = \frac{mr^2sx}{(s + x)^3}$$

(6) After analysis (by quotient rule), we realize that

$$x = \frac{s}{2}$$

is the position of the bullet before which force increases (if x < s/2 then $f'(x) > 0$) and after which force decreases. Thus the two dimensional version of this one dimensional representation of the relation between force and position is:
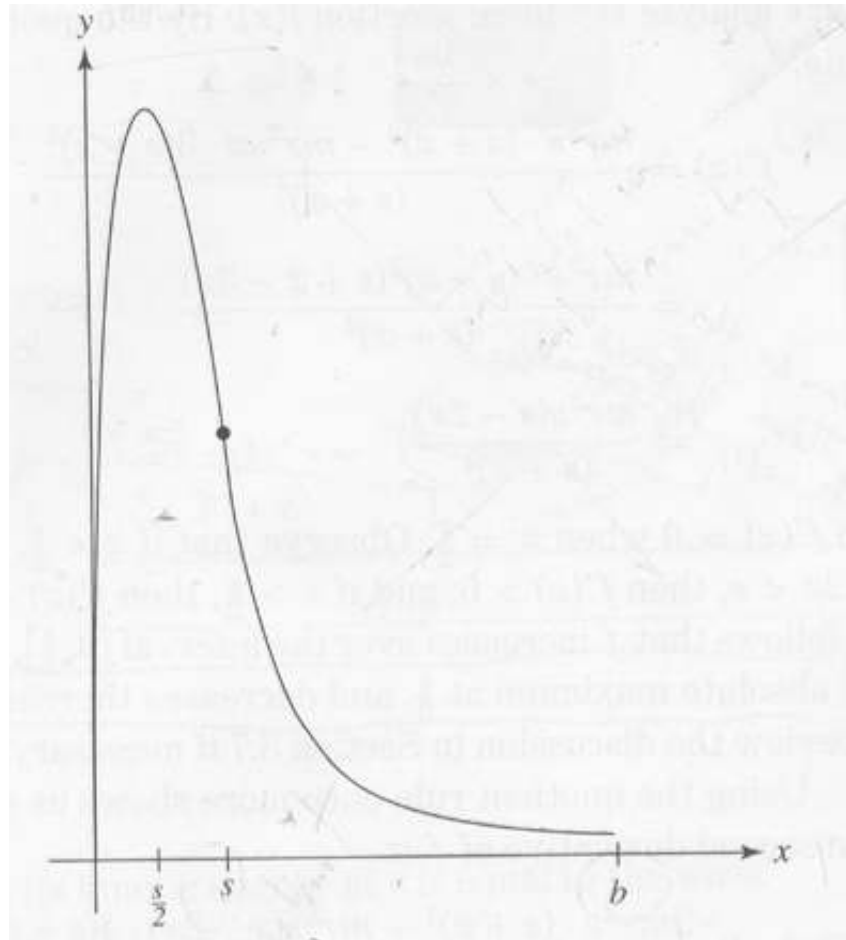
y = force; x = position; b = length of gun; s = constant

This is the general shape. Its specific shape will depend on the *ordination* (numerical cutting up) of the x and y coordinate, i.e. on the value of the constant r and s.

Now that the quantitative representation (*structure*) of the whole phenomenon of firing of the bullet (inside the shell) is at hand (explosion of powder ---> force generated ---> velocity of the bullet, within the parameter set by the characteristics of the barrel and the bullet), "prediction" can be made about any *moment* of the motion of the bullet (another cross-section of the whole in any of its aspect) to help the design of the gun. Knowing the pressure generated in the barrel would help further.

k = diameter of the circular cross-section of the barrel, or *caliber*. Area of this cross-section is:

$$\Pi \left(\frac{k}{2}\right)^2 = \frac{\Pi k^2}{4}$$

Force can be calculated, in another way, as pressure x area. "Pressure" is simply one of the constituents into which force is analyzed -- but now in another way: the first way, into mass and acceleration, is from the point of view of the moving shell; the second way, into pressure and area, is from the point of view of the barrel. Force is now:

$$f(x) \text{ [force at position x]} = \frac{\Pi k^2}{4} p(x)$$

$$\text{and } p(x) \text{ [pressure at position x]} = \frac{4}{\Pi k^2} f(x)$$

The two dimensional quantitative representation of pressure (the pressure curve) is a curve of the same general shape as that of f(x). Pressure (at the position of the shell) rises rapidly before and reaches maximum at s/2, after which it drops, rapidly at first, then gradually.

Now the quantitative (exact) representation of the motion of bullet in the barrel is complete, a gun can be designed. One can of course make a gun with mere qualitative representations ("make the barrel about the length of that table, and the diameter of barrel the same as the coin...") but the imprecision is likely to result in a gun barely working and dangerous. Quantitative representation not only allows one to grasp the structure of reality with greatest clarity because of its exactitude, its advantage in engineering is usually more familiar.

To analyze the phenomenon into 2 constituents, map the 2 constituents on to the 2 dimensions of a 2-D surface, grid the surface and so the phenomenon (numeralize the coordinate), and, when with such grid it becomes possible to translate the 2-D graph into 1-D algebra, then find the relationship between the constituents, which becomes the structure of the phenomenon: this is what the Cartesian Revolution in mathematics enables.

It should be noted that in this process of capturing the structure of the phenomenon, in this process of dissection and recomposition, and also in this process of dimensional reduction, new categories of thought/ perception are generated (in the case of motion: "velocity", "acceleration", "force", "pressure") which grid the phenomenon *also qualitatively* in a new way, under new light -- in a way not only to help us comprehend, through quantitative representation, the phenomenon precisely, in its structure, in its essence, but also to enrich our qualitative awareness of the world. When we think about motion, as when we drive a car, we have more vocabulary ("this car is not only fast, but it accelerates fast... during accident, however, its force of impact could break through a wall 2 feet thick") to cut open the phenomenon and to comprehend it more deeply; this is in contrast to people 2,000 years ago riding on chariot, whose limited vocabulary concerning motion did not allow them to comprehend the movement of the chariot

in similar depth.

As said, calculus is the analyzation of a phenomenon into its two constituent elements, and the representation, through mathematical equations, of the relationship between the two constituents (either multiplication [integration] or division [differentiation]). This relationship between the two constituents is the structure of the phenomenon, and its representation or calculation is possible because it is always non-arbitrary, i.e. there is always between the constituents a definite relationship which is forever constant in its non-arbitrariness. A motion of a thing is analyzed into position and time both of which increase or decrease in a definite, non-changing proportion with one another: and this proportion is the structure. This is why it is possible to "calculate" velocity (i.e. to deduce from what are already represented, i.e. distance travelled and time elapsed, another aspect of the phenomenon which is not yet represented, velocity), and on the basis of velocity, acceleration.

The relationship between the constituents is non-arbitrary because the whole of which they are parts (the motion) is a unity -- this much is obvious. But the whole Universe is an unity, so that this amount of force will always produce this amount of motion of this amount of mass which will trace out a path in this definite manner. This is why we can find laws of nature, i.e. the non-changing, non-arbitrary relationship among the parts of the Universe; e.g. Newton's law Force = mass x acceleration. These laws of nature in equations are our representation of the order of the Universe.

Any "formula" is an exact quantitative description of the relationship between the components of a whole, so that, in the case of Le Duc's interior ballistics, the velocity and position of the shell inside the barrel are related ever constantly as:

$$v(\mathrm{x}) = \frac{\mathrm{rx}}{\mathrm{s} + \mathrm{x}}$$

as we have seen above. Acceleration is then deduced therefrom, and then force, because the Newtonian formula should always be valid insofar as the Universe is a whole (so that this amount of force will always produce this amount motion...). Prediction, again, is possible because the relationship between the constituents is non-arbitrary and so the structure of the whole, and in this way the representation of the relationship will allow us to represent another cross-section of the total phenomenon of the movement of the bullet: e.g. muzzle velocity.

For example, the Le Duc specification of the springfield rifle is:

$$v(\mathrm{x}) = \frac{3716\mathrm{x}}{0.65 + \mathrm{x}}$$

Its muzzle velocity is "predicted" as:

$$v(2) = \frac{(3716)(2)}{} = \text{(approximately) 2800ft. per sec.}$$

$$0.65 + 2.00$$

given that the barrel = 2 feet long and its various characteristics are determined as, in terms of r and s, 3716 and 0.65.

Note these analyzations of phenomenon:

- work = force * distance (and so determined via integration) because work = sum of all forces, each force = force over an infinitesimal interval of distance;
- force = pressure * area, because force = sum of all pressures, each pressure = pressure over a small area of the total area acted upon by force.

ACADEMY | previous section | Table of Content | next section | GALLERY