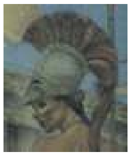


Scientific Enlightenment, Div. 2

A. 4: The Problem of Representation (and the Constitution of Classical Mechanics)

Chapter 9: The Relativity Theory of Albert Einstein

9.1. Special relativity

In accordance with our model of the history of science as an exemplification of Plato's allegory of the cave, and of science in general and physics in particular as the most advanced eidetic study, we have interpreted the formulas and equations of classical mechanics as the *eidoi* that make possible the showing of the mechanical (i.e. present-at-hand) aspects of our everyday world. But so far classical mechanics -- the quantitative representation of the motion of macroscopic objects (or "the kinematics of rigid objects" [die kinematik des starren Körpers]), that is, objects of everyday perception -- is constructed from such quantitative representation of motion (or of a whole system of motions by the time of Lagrange) from a single perspective -- from a single rigid body of reference or coordinate system -- and is "subjective" in this sense (although the physicists themselves are under the illusion that their construction is valid in all perspectives, in all possible coordinate systems: see below). This limitation is noticed only when the representation of motion from one perspective cannot really be reconciled with the representation of the same motion from another perspective. In such conflict, the meanings of space and time -- as the representation of their combined effect, motion as "space traversed during time elapsed", or "space divided by time", breaks down in that different perspective -- seem to fade away. Hints of this exist already in the everyday experience accessible to classical mechanics:

> Es ist unklar, was hier unter "Ort" und "Raum" zu verstehen ist. Ich stehe am Fenster eines gleichförmig fahrenden Eisenbahnwagens und lasse einen Stein auf den Bahndamm fallen, ohne ihm einen Schwung zu geben. Dann sehe ich (abgesehen vom Einfluss des Luftwiderstandes) den Stein geradlinig herabfallen. Ein Fußgänger, der die Übeltat vom Fußwege aus mit ansieht, bemerkt, daß der Stein in einem Parabelbogen zur Erde herabfällt. Ich frage nun: Liegen die "Orte", welche der Stein durchläuft, "in Wirklichkeit" auf einer Geraden oder auf einer Parabel? (1916, p. 6)
>
> It is not clear what is to be understood here by "position" and "space." I stand at the window of a railway carriage which is travelling uniformly, and drop a stone on the embankment, without giving it any swing. Then (disregarding the influence of the air resistance) I see the stone descend in a straight line. A pedestrian who observes the misdeed from the footpath notices that the stone falls to earth in a parabolic curve. I now ask: Do the "positions" traversed by the stone lie "in reality" on a straight line or on a parabola?

But these thus far fail to disturb classical mechanics. As Kuhn has noted, anomalies or failures in prediction are generally long recognized before they finally transit into the new paradigm-prefiguring "crisis." But, the inducement to such crisis is about to be triggered by phenomena from another branch of mechanics than kinematics: electromagnetism and optics. It is then that "the upward movement out of the cave" properly begins, when the *eidoi* hitherto arrived at are shown to be merely for the showing of shadows on the cave wall -- and the "space" and "time" of everyday world merely shadows -- cast by some higher reality behind that is "more real": a different structure of "spacetime." In other words, the *eidoi* of classical mechanics are themselves shadowy distortions of some other, more *real eidoi*. Positively speaking, unlike before, the new paradigm this time is of a world completely beyond the possibility of our natural sense-perception.

More than 2,000 years ago, Zeno's paradoxes of motion have already shown that our common-sense notion of space and time has to be false (and that a quantized space necessarily implies a limiting velocity and the relativity of time); hence, it is simply expected that classical mechanics, insofar as it is the quantification of this common-sense without transcending it, will have inconsistencies within it surface to explicit level at some point in its quantification project.

The apparent contradiction between Newtonian mechanics and Lorentz-Maxwell's electromagnetism, and Einstein's solution. So, the special theory of relativity arose from the attempt to resolve the apparent conflict between the principle of relativity derived from the classical mechanics of the motion of macroscopic objects (the kinematics of rigid bodies) and the Maxwell theory of electromagnetism, a conflict which had become especially acute by 1900. What were involved in this contradiction in the picture of the physical world at the time were three principles: firstly the two apparent principles that drew all the attention, the principle of relativity bestowed by classical mechanics and "the law of the propagation of light in vacuum" that came from the electrodynamics of Maxwell; and then the one hidden, unspoken principle that everyone overlooked, Newton's principle of absolute space and absolute time.

The principle of relativity arises from the impossibility of determining the state of motion of an inertial frame by means of mechanical experiments carried out within a closed system with center of mass at rest in this frame (CP, TR, p. 255). This is because an experiment in mechanics (the measurement and representation of the motion of some macroscopic objects) always gives the same result, whether in a stationary laboratory on earth or in a laboratory moving in a straight line and at a constant speed (relative to the stationary one), as Galileo has demonstrated long ago. It is thus sometimes called Galilean relativity. It is for this reason that you can drink your coffee in the airplane as long as it flies in a straight line and at a constant speed.[1] In Einstein's formulation:

> Ist K' ein in bezug auf K gleichförmig und drehungsfrei bewegtes Koordinatensystem, so verläuft das Naturgeschehen in bezug auf K' nach genau denselben allgemeinen Gesetzen wie in bezug auf K (1916, p. 8 - 9).
>
> If, relative to K, K' is a uniformly moving co-ordinate system devoid of rotation, then natural phenomena run their course with respect to K' according to exactly the same general laws as with respect to K.

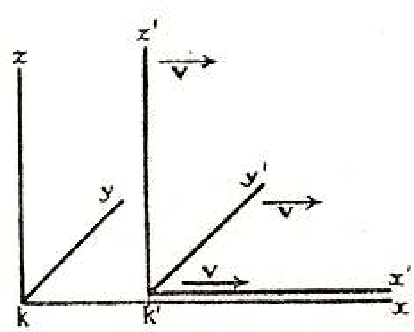
Thus:

> Für die physikalische Beschreibung der Naturvorgänge ist keiner der Bezugskörper K, K' vor dem anderen ausgezeichnet (1916, p. 41).
>
> For the physical description of natural processes, neither of the reference bodies K, K' is preferable [lit. "specially marked out"] as compared with the other.

This can be summarized in another way by saying that "the state of rest and the state of uniform and rectilinear motion do not differ." (LR, p. 20) This also means that you cannot tell whether the train is moving on a stationary earth or the earth is moving beneath a stationary train. The Galilean character of this principle of relativity lies in the qualification for this case that, firstly, the relative motion one coordinate system or reference body has with respect to another must be uniformly rectilinear and non-rotary ("straightline, at constant speed", as mentioned: geradlinig gleichförmige, rotationsfreie), which kind of movement is called "uniform translation" (gleichförmige Translationsbewegung: "'gleichförmig', weil von konstanter Geschwindigkeit und Richtung, 'Translation', weil der Wagen relativ zum Fahrdamm zwar seinen Ort ändert, aber hierbei keine Drehungen ausführt"; 1916, p. 8; "uniform translation movement: 'uniform' because it is of constant velocity and direction, 'translation', because although the carriage changes its position relative to the embankment yet it does not rotate in so doing'); and that, secondly, the coordinate system or reference body must be Euclidean. Einstein thus refers to it as the principle of relativity "in the restricted sense" (im engeren Sinne), and the theory of relativity that is to be built upon it as "special", because uniform translation motion is a special case of motions in general, just as (we saw earlier) the circle is a special case of ellipses. Consciousness always starts with the provincial, special case, a limited, easiest portion of the whole reality, before it moves on to comprehend the more difficult and complex total picture of which the special case makes up only a small part.

These Euclidean coordinate systems in uniform translation are also called the "inertial frames" or the "inertial coordinate systems", since they are essentially characterized by the fact that they are those in which Newton's first law, the law of inertia, holds.

The subjective nature of classical mechanics had never been noticed because it is thought that the transformations attached to the (Galilean) principle of relativity (below) had actually allowed it to account for all inertial frames. The admission of all inertial frames usually takes the form of such problem: how to find the place and time of an event in relation to the train, when we know the place and time of the event with respect to the railway embankment (the train moving at a uniform speed and in a straight line relative to the embankment)? Marked out with each of (the perspectives of) the train and the embankment are imagined three surfaces perpendicular to each other, designated as "co-ordinate planes" ("co-ordinate system"). A co-ordinate system K then corresponds to the embankment, and a co-ordinate system K' to the train. An event, wherever it may have taken place, would be fixed in space with respect to K by the three perpendiculars x, y, z on the co-ordinate planes, and with regard to time by a time value t. Relative to K', the same event would be fixed in respect of space and time by corresponding values x', y', z', and t', which of course are not identical with x, y, z, t. To formulate the problem again more precisely: What are the values x', y', z', t', of an event with respect to K', when the magnitudes x, y, z, t, of the same event with respect to K are given? (1916, p. 21 - 2)



In the framework of classical mechanics this kind of problem in Galilean relativity -- to know what the Cartesian spatial and temporal coordinates of an event that is seen from one perspective (one coordinate system) are in another perspective that is in uniform translation motion relative to the first -- is solved with the method of *Galilean transformation*:

$$x' = x - vt$$
$$y' = y$$
$$z' = z$$
$$t' = t$$

Reversely understood this is just our *common sense theorem of the addition of velocities*. For example, imagine a light ray emitted along the embankment (along the x axis of K) from its coordinate origin (x = 0): while it would reach, after time t, x = ct within the coordinate frame (or perspective) of the embankment, within that of the train (K') it would reach after time t: x' = ct - vt = (c - v)t, which is what our common-sense manner of calculating the velocity of light c as seen from the moving train (relative velocity being derived from the simple combination of the two velocities: c- v) would predict. Note that *this addition of velocities presupposes that space and time were absolute* (the unspoken third principle in question here).

The second principle, the constancy of the speed of the light in vacuum irrespective of the velocity of its source, not to be confused with *the constancy of the speed of light irrespective of the direction and velocity of the motion of the observer*, emerges from Maxwell's electrodynamics. Rynasiewicz (p. 40) stresses that Einstein's first, 1905 (a) paper on special relativity -- that century-making event -- referred only to the former, by then well-established principle: "Each light ray travels in the 'stationary' coordinate system with the definite velocity [c], independently of whether this light ray is emitted from a stationary or a moving body" ("Jeder Lichtstrahl bewegt sich im 'ruhenden' Koordinatensystem mit der bestimmten Geschwindigkeit V [V = c, or 300,000 km per second], unabhängig davon, ob dieser Lichtstrahl von einem ruhenden oder bewegter Körper emittiert ist", 1905, p. 895): "the law of the propagation of light in vacuum" (das Gesetz der Lichtausbreitung im Vakuum, 1916, p. 12). In this statement, "the 'stationary' coordinate system is an inertial frame, which the contemporary reader is naturally invited to identify with the rest frame of the ether [see below]. [This] light postulate is in fact a trivial consequence of Maxwell's theory, or of any wave theory of light based on the assumption of a stationary ether, and thus a completely uncontroversial assumption for Einstein's contemporary reader." And yet, today so many textbooks erroneously state that Einstein had in mind for his "law of the propagation of light in vacuum" the fact that light travels at the same velocity for all inertial frames ("the constancy of the speed of light irrespective of the direction and velocity of the motion of the observer") (R, 39 - 40). "This has engendered the myth that Einstein created the special theory of relativity in direct response to the null result of [the Michelson-Morley] experiment" (ibid., p. 40). Rather, Einstein's concern was with the justification of the principle of relativity with which this light postulate seemed to be in conflict, as it seemed in conflict with Galilean transformation.

Originally, Maxwell's equations were derived as valid for the inertial frame at rest. The attempt to apply them to moving bodies since Maxwell had proven inadequate. The application of Galilean transformation to his equations in the case of electromagnetism from moving sources resulted in different laws for electrodynamics, thereby violating the principle of relativity. The speed of electromagnetic wave from moving sources remained c, without the addition of velocities. This by itself did not pose a problem within the picture of the physical universe underlying the Maxwellian electrodynamics: Although Maxwell's electrodynamics had introduced something alternative to classical mechanics' world as consisting of rigid objects -- the notion of "field" -- at first the conception of the field was such that it occurred only within ponderable masses, that it only described a physical state of matter (einen Zustand der Materie) (1916, p. 93). For example, the field-motion of liquid, or heat conduction in a solid body. Since Maxwell's electrodynamics was essentially a theory of electromagnetism and light as wave- or field-like, it was natural to consider light as, just like other fields, the vibratory state of some material. In other words, a "medium" was necessary for the propagation of light waves, and this medium was considered different than the ponderable matter for the other fields, because light did propagate in empty space. (Sound, for example, as the vibratory state of air, cannot travel in vacuum.) An "ether", "luminiferous ether" (Lichtmedium), was therefore assumed to permeate all of space to serve as the medium for light. It can then be seen why the constancy of the speed of the light irrespective of the velocity of its source was a trivial consequence of the this wave conception of light. The velocity c of light and the velocity v of the moving body emitting it needn't be combined (for the observer at rest) because light was propagated with respect to the ether at rest, not with respect to its source, just like sound: "Sound waves from a passing train, for example, travel at the velocity of sound compared to the air, whether the train is motionless or moving at 100 m/s." (J. W.) A bullet, however, fired from a fighter plane has a velocity (for the observer at rest) which is the combination of its velocity and that of the plane. The question then arose as to what physical characteristics this medium of which light was the vibration had, and whether this medium moved with (was dragged along by) the ponderable bodies that emitted light, or remained immobile relative to it. This was the problem of "the electrodynamics of moving bodies or in moving media."

Fizeau's experiment was the first definitive statement on this matter. As light traveled in a motionless liquid with a velocity v, the liquid was made flowing through the tube, and, since the velocity of the liquid w and the velocity of light in liquid v were known, the experiment tested the velocity of light relative to the tube. In the case of the sound, if we place the source of sound in a moving carriage in which all windows and doors are shut tight, we shall discover that the sound travels at the same speed in all directions, because its medium, air, is sealed -- dragged along -- in the carriage and so unaffected by its movement. If the cage is merely made of thin rods, the speed of sound will be less in the direction of the motion of the carriage. (LR, p. 31) If ether could be dragged along in the same manner within the liquid, then (given the assumption of absolute space and time) the movement of the liquid should result in the speed of light relative to the tube being $W = w + v$ or $w - v$. Rather, Fizeau found that $W = w + \alpha v$, where "α is a number understood to be between 0 and 1, and depends on the index of refraction n: $\alpha = 1 - (1/n^2)$" (1910, p. 9). This disproved the movement of the ether along with the liquid. But "if the ether is not dragged with matter, it should be possible to detect motion relative to a reference frame fixed in the ether by means of optical experiments. However, all attempts to detect the motion of the earth through the ether by optical experiments failed" (CP TR, p. 255). The most famous of these attempts was the Michelson-Morley experiment, whose significance will be commented on shortly.

By this time Hendrik Antoon Lorentz was formulating the Maxwell-Lorentz electromagnetic theory. Lorentz, the "unconditioned follower of the atomic theory of matter" (1920), in 1895 attempted to re-incorporate into Maxwell's theory "an important element of the pre-Maxwellian continental tradition", the "atoms of electricity" (CP TR, p. 256). This picture of the physical universe consisted of ether pervading all regions of space and remaining absolutely at rest, and the ponderable matter moving or resting in it, which was composed of elementary particles that, at least in part, were provided with electric charges (1910, p. 10). Matter only influenced the ether through these charged elementary particles, which created the electromagnetic fields according to Maxwell's equations. And the ether only acted on the ponderable matter through the electric and magnetic forces that these fields exerted on its charged particles. The most significant element in this worldview was that the ether defined an absolute rest frame, since neither did it take part in the movement of the ponderable masses, nor did its parts have any movement relative to one another. (1916, p. 94) With regard to Fizeau's experiment Lorentz noted that "a uniform translation of the velocity v of the equipment in relation to ether" did have an influence on the observable speed of light, but this influence was too small for detection, being in the order of $(v/c)^2$. He developed the theorem of the corresponding states "for generating solutions for a moving system of bodies -- lenses, apertures, prisms, or whatever -- from solutions for a corresponding system of stationary bodies. The theorem works by exploiting a first-order formal covariance that results from substituting into Maxwell's equations, expressed in moving coordinates, auxiliary variables for the field quantities, which for the case of free space reduce to":

$$E' = E + \frac{1}{c}(v \times H)$$

$$H' = H - \frac{1}{c}(v \times E)$$

where E, E' and H, H' are field components measured in the rest system; "and the variable, called 'local time'":

$$t' = t - \frac{1}{c^2}(v_x\xi + v_y\eta + v_z\zeta)$$

"where t is the true time, (v_x, v_y, v_z) the velocity components of the moving system, and (ξ, η,ζ) the spatial coordinate relative to the moving system." (R, p. 51) By means of these *eidoi*, Lorentz was able to predict the null results (of the motion of ether) of all experiments up to the first order. But then came that famous Michelson-Morley experiment of 1887 ("On the Relative Motion of the Earth and Luminiferous Ether", *The American Journal of Science*, Nov. 1887), which was a different case. We have all heard of this experiment which, taking the earth as a moving body with a velocity of 30 km/sec (the velocity of its rotation around the sun), arranged light beams and mirrors to measure the speed of light moving in the same direction as the Earth through space and of light in the opposite direction. ("The earth may well be considered to be moving uniformly along a straight line in that infinitesimal fraction of a second which it takes light to pass through the points of observation...." LR, p. 29 - 30) The significance of this experiment was two-fold. Firstly, although the velocity of light didn't depend on the velocity of its source because of the rest frame of the ether, it should depend on the velocity of the observer. After all, if the observer (on the moving earth) was moving through the ether with a velocity v toward an oncoming light beam, and light was moving toward him at velocity c, then the observer should see the light moving at an apparent rate, compared to his coordinate system, of c + v (the theorem of the addition of velocities). Secondly, this experiment was designed specifically to detect the second order difference. And yet no movement of the earth relative to the ether was detected, since the speed of light remained constant no matter from which direction the observer measured it. Given the assumption about absolute space and time, the non-detection of the addition of velocities (the failure of the *eidos* of Galilean transformation) meant that the earth (the observers on it) was not moving with respect to the ether. Unless the laboratory had encased the ether and so dragged it along. But this contradicted the results of Fizeau's experiment. Together, they put the existence of ether in jeopardy. In order to save the ether hypothesis, Lorentz and Fitzgerald were forced to admit that "any body in movement in relation to the ether shortens itself in the direction of the movement by a fraction equal to $1/2(v/c)^2$ or -- what amounts to the same thing when one only considers the terms of the second order -- that the length of the body is diminished in this direction by the proportion" (1910, p. 14):

1 :

$$\sqrt{1 - (v^2 / c^2)}$$

This is just the sufficient amount to compensate for the difference in time of light's travel. In other words, while light may indeed transit slower or faster on a moving media, it ended up travelling a shorter or longer distance or during a shorter or longer time that exactly canceled out the drift. In Lorentz's picture of the physical universe, it was the electrons that contracted in the direction of their motion, and since "the intermolecular forces responsible for the equilibrium state of macroscopic rigid bodies are ultimately electromagnetic in origin, such contraction factor is to be expected". This thinking gave what Poincaré would later name the Lorentz transformation, as the substitute for Galilean transformation, taking into account the "contraction" under such circumstance:

$$x' = \frac{x - vt}{\sqrt{1 - \frac{v^2}{c^2}}}$$
$$y' = y$$
$$z' = z$$
$$t' = \frac{t - \frac{v}{c^2} \cdot x}{\sqrt{1 - \frac{v^2}{c^2}}}.$$

The underlying physical picture of the universe that the *eidoi* embodied by particular phenomena (in experiments) would point to was thus saved: absolute space and absolute time, ether, and electrons. Lorentz's theory, however, seemed to conflict with the principle of relativity. As Einstein summarizes:

> Die klassische Mechanik... lehrt die Gleichwertigkeit aller Inertialsysteme (bzw. Inertialräume) für die Formulierung der Naturgesetze (Invarianz der Naturgesetze in bezug auf den Übergang von einem Inertialsystem auf ein anderes). Die elektromagnetischen und optischen *Experimente* lehrten dasselbe mit erheblicher Genauigkeit. Aber das Fundament der elektromagnetischen *Theorie* lehrte die Bevorzugung eines besonderen Inertialsystems, nämlich das des ruhenden Lichtäthers. Diese Auffassung des theoretischen Fundamentes war gar zu unbefriedigend. (1916, p. 95)
>
> Classical mechanics... taught the equivalence of all inertial frames (or inertial spaces) for the formulation of the laws of nature (the invariance of the laws of nature in respect to the transition from one inertial system to another). The electromagnetic and optical *experiments* taught the same thing with considerable precision. But the foundation of electromagnetic *theory* taught the preference for a particular inertial system, namely that of the luminiferous ether at rest. This conception of the theoretical foundation was much too unsatisfactory.

An example of the experiments of which Einstein speaks here is that to which he points in the opening of his 1905 paper, that which concerns the creation of an electric current by the relative motion of a magnet and a conductor. In electrodynamics, the stationary ether has defined an absolute rest frame or coordinate system for the entire universe. Therefore according to the customary understanding of Maxwell's electrodynamics, the current produced is different depending on which of the two is absolutely at rest. If the magnet moves and the conductor is at rest, there arises an electric field with a definite associated energy, which then produces the current. In contrast, if the magnet is at rest and the conductor moves, there should be no electric current, but only an "electromotive force" with no corresponding energy. But, in fact, i.e. in experiment, the intensity and direction of the electric current are the same in both cases, given the same relative motion of the magnet and the conductor. This shows Einstein that the principle of relativity must be a universal principle applicable in electrodynamics as well, and therefore that "no phenomenal properties correspond to the idea of absolute rest, not only in mechanics, but also in electrodynamics, but that the same electrodynamic and optical laws hold for all coordinate systems for which the equations of mechanics hold...." (1905a, p. 891; "... daß dem Begriffe der absolute Ruhe nicht nur in der Mechanik, sondern auch in der Elektrodynamik keine Eigenschaften der Erscheinungen entsprechen, sondern daß vielmehr für alle Koordinatensysteme, für welche die mechanischen Gleichungen gelten, auch die gleichen electrodynamischen und optischen Gesetze gelten...") Thus, Lorentz's ether which blocks the entry of the principle of relativity into electrodynamics and which can't be measured at all must go, even though his *eidos* (his transformation) seems essentially correct -- c is stabilized as constant, that is, experimental facts are harmonized with electrodynamics. This means that the content of the *eidos* -- the constancy of the speed of light irrespective of the direction and velocity of the observer, the contraction of length and time dilation -- must be the *effect* of some other, more simplified reality. Einstein therefore gives up the ether and questions directly the third "unspoken" principle, that of absolute space and absolute time (1905a, p. 895), in order to preserve the relativity principle and derive *the constancy of the speed of light in vacuum irrespective of the velocity of its source* not from the ether at absolute rest but from a new physical picture of the universe, a new conception of space and time.

> Instead of regarding the failure of electromagnetic and optical experiments to detect the earth's motion through the ether as something to be deduced from the electrodynamical equations [Lorentz's approach], he took this failure as empirical evidence for the validity of the principle of relativity in electrodynamics and optics.... Einstein now confronted the problem of making Maxwell-Lorentz electrodynamics compatible with the principle of relativity. He did so by means of a principle drawn from electrodynamics, the principle of the constancy of the velocity of light.... Einstein dropped the ether from consideration, and took the constancy of the velocity of light as a second postulate supported by all the empirical evidence in favor of the Maxwell-Lorentz theory. When combined with the relativity principle, this leads to an apparently paradoxical conclusion: the velocity of light must be the same in all inertial frames [*the constancy of the speed of light irrespective of the direction and velocity of the observer*]. This result conflicts with the Newtonian law of addition of velocities [or Galilean transformation], forcing a revision of the kinematical foundations of electrodynamics. (CP TR, p. 257)

The constancy of the speed of light in vacuum irrespective of the velocity of its source is the *result* of the principle of relativity, which also happens to explain the constancy of the speed of light irrespective of the direction and velocity of the motion of the observer. In fact, the constancy of the speed of light in vacuum irrespective of the velocity of its source is a special case of the constancy of the speed of light irrespective of the direction and velocity of the observer. In Fizeau's experiment, for example, if there were an observer inside the tube moving with the water, s/he would measure the same speed of light as does the stationary observer outside the tube. (The water can be considered stationary, and the observer outside the tube, moving.) The contradiction between Fizeau's experiment and that of Michelson and Morley is resolved. Insofar as the addition of velocities or Galilean transformation with which this new derivation of the law governing the propagation of light conflicts is the result of absolute space and absolute time, Einstein attempts to clear this obstacle through a logical analysis of the concept of "time", in order to expose the logical contradiction inherent in the concept of absolute time, thereby getting rid of the absoluteness of space and time -- and so absolute rest -- which makes electrodynamics incompatible with the principle of relativity.

An example may illustrate the problem with *this third principle, the absoluteness of space and of time*.

Es sei ein ruhender starrer Stab gegeben; derselbe besitze, mit einem ebenfalls ruhenden Maßstabe gemessen, die Länge *l*. Wir denken uns nun die Stabachse in die X-Achse des ruhenden Koordinatensystems gelegt und dem Stabe hierauf eine gleichförmige Paralleltranslationsbewegung (Geschwindigkeit v) längs der X-Achse im Sinne der wachsenden x erteilt. Wir fragen nun nach der Länge des *bewegten* Stabes, welche wir uns durch folgende zwei Operationen ermittelt denken:

Let there be given a stationary rigid rod; and it possesses the length *l* as measured by a stationary measuring-rod. We now imagine the axis of the rod lying along the x axis of the stationary coordinate system, and imparting the rod a parallel uniform translation movement (of velocity v) along the x axis in the direction of increasing x. We ask for the length of the *moving* rod, which we imagine can be ascertained through the following two operations:

a.) Der Beobachter bewegt sich samt mit dem vorher genannten Maßstabe mit dem auszumessenden Stabe und mißt direkt durch Anlegen des Maßstabes die Länge des Stabes, ebenso, wie wenn sich auszumessender Stab, Beobachter und Maßstab in Ruhe befänden.

a.) The observer moves together with the given measuring-rod and the rod to be measured, and measures the length of the rod directly by superposing the measuring-rod, just as if the rod being measured, the observer, and the measuring rod were at rest.

b.) Der Beobachter ermittelt mittels im ruhenden Systeme aufgestellter... synchroner, ruhender Uhren, in welchen Punkten des ruhenden Systems sich Anfang und Ende des auszumessenden Stabes zu einer bestimmten Zeit t befinden. Die Entfernung dieser beiden Punkten, gemessen mit dem schon benutzten, in diesem Falle ruhenden Maßstabe ist ebenfalls eine Länge, welche man als "Länge des Stabes" bezeichnen kann....

b.) By means of stationary, synchronized.... clocks set up in the stationary system, the observer ascertains at what points of the stationary system the beginning and the end of the rod to be measured are located at a definite time. The distance between these two points, measured by the measuring-rod already employed, which in this case is at rest, is also a length which one may designate as "the length of the rod"....

Die allgemein gebrauchte Kinematik nimmt stillschweigend an, daß die durch die beiden erwähnten Operationen bestimmten Längen einander genau gleich seien, oder mit anderen Worten, daß ein bewegter starrer Körper in der Zeitepoche t in geometrischer

The generally employed kinematics tacitly assumes that the lengths determined by the two operations mentioned are precisely equal to one another, or in other words, that a moving

Beziehung vollständig durch *denselben* Körper, wenn er in bestimmter Lage *ruht*, ersetzbar sei. (1905, p. 895 - 6)

rigid body at time t in geometric respects is fully replaceable by the *same* body when it is *at rest* in a definite place.

This last assumption is the consequence of absolute space and absolute time, the Galilean transformation, and the theorem of the addition of velocities, which, altogether, entail classical mechanics' (and our common sense's) assumption about the absoluteness of the simultaneity of events -- that events can *really* be simultaneous. "Die Gleichzeitigkeit zweier bestimmter events in bezug auf ein Inertialsystem involviert die Gleichzeitigkeit dieser events in bezug auf alle Inertialsysteme. Dies ist gemeint, wenn man sagt, die Zeit der klassischen Mechanik ist absolut." (1916, p. 96; "The simultaneity of two definite events in respect to one inertial system involves the simultaneity of these events in respect to all inertial systems. This is what is meant, when one says that the time of classical mechanics is absolute"; RSGT, p. 170.) Therefore, to show the compatibility between the constancy of speed of light in vacuum and the principle of relativity, to derive the former from the latter, to clear away absolute time, Einstein demonstrates the untenability of the absolute simultaneity of events, and then derives the Lorentz transformation from the analysis of the resultant relativity of time and length, thus showing that this *eidos* in fact points to the contraction of space itself and the dilation of time itself for the moving body *as appear in the rest frame* (for the moving body itself, from its own perspective, experiences no such contraction and dilation), rather than an actual contraction of the spatial extension of the moving body and the actual dilation of its local time within the milieu of an absolute, "objective" common time.

Simultaneity in fact constitutes the essence of the meaning of time. Just as in the previous skepticism about the *real* meaning of space, we seem to be at a loss about what "time" means *really*, that is, how to measure time precisely, even in our own perspective (in one inertial frame), when we are so asked. "Der Begriff existiert für den Physiker erst dann, wenn die Möglichkeit gegeben ist, im konkreten Falle herauszufinden, ob der Begriff zutrifft oder nicht." (1916, p. 14; "The concept exists for the physicist only when the possibility is given of finding out in concrete circumstances whether the concept is fulfilled or not"). In other words, the meaning of time can only be ascertained through an experimental or operational procedure of measuring it, and, in this way, any operational determination of the meaning of time in fact involves a simultaneity of events -- just that synchronization (the simultaneous running) of two clocks in the previous operation (b) of the measurement of space. Simultaneity in time is the key to the measurement of both time and space, such that both space and time depend on the validity of simultaneity in retaining the objective, perspective-free meaning that classical mechanics attributes to them. As Einstein explains in his 1905 (a) seminal paper (p. 893):

Wir haben zu berücksichtigen, daß alle unsere Urteile, in welchen die Zeit eine Rolle spielt, immer Urteile über *gleichzeitige Ereignisse* sind. Wenn ich z. B. sage: "Jener Zug kommt hier um 7 Uhr an," so heißt dies etwa: "Das Zeigen des kleinen Zeigers meiner Uhr auf 7 und das Ankommen des Zuges sind gleichzeitige Ereignisse."

We have to take into account that all our judgments in which time plays a part are always judgments of *simultaneous events*. If I for instance say, "That train arrives here at 7 o'clock," this means something like: "The pointing of the small hand of my watch to 7 and the arrival of the train are simultaneous events."

Es könnte scheinen, daß alle die Definition der "Zeit" betreffenden Schwierigkeiten dadurch

It would seem that all the difficulties concerning the überwunden werden könnten, daß ich an Stelle der "Zeit" die "Stellung des kleinen Zeigers meiner Uhr" setze. Eine solche Definition genügt in der Tat, wenn es sich darum handelt, eine Zeit zu definieren ausschließlich für den Ort, an welchem sich die Uhr eben befindet; die Definition genügt aber nicht mehr, sobalt es sich darum handelt, an verschiedenen Orten stattfindende Ereignisreihen miteinander zeitlich zu verknüpfen, oder... Ereignisse zeitlich zu werten, welche in von der Uhr entfernten Orten stattfinden.

definition of time can be overcome by my substituting "the position of the small hand of my watch" for "time". In fact such definition is satisfactory when we are concerned with defining a time exclusively for the place where the watch is located; the definition is however no longer satisfactory as soon as we are concerned with connecting in time events occurring at different places with one another, or... with evaluating the times of events occurring at places remote from the watch.

Wir könnten uns allerdings damit begnügen, die Ereignisse dadurch zeitlich zu werten, daß ein samt der Uhr im Koordinatenursprung befindlicher Beobachter jedem von einem zu wertenden Ereignis Zeugnis gebenden, durch den leeren Raum zu ihm gelangenden Lichtzeichen die entsprechende Uhrzeigerstellung zuordnet. Eine solche Zuordnung bringt aber den Übelstand mit sich, daß sie vom Standpunkte des mit der Uhr versehenen Beobachters nicht unabhängig ist, wie wir durch die Erfahrung wissen. Zu einer weit praktischeren Festsetzung gelangen wir durch folgende Betrachtung.

We might in any case content ourselves with determining the time values of events by the operation that the observer stationed together with the clock at the origin of the coordinates coordinates the corresponding positions of the hands of the clock with the light signals given out by every event to be timed, and reaching him through empty space. Such coordination brings with it however the disadvantage that it is not independent of the standpoint of the observer furnished with the clock, as we know from experience. We arrive at a much more practical determination through the following consideration.

Befindet sich im Punkte A des Raumes eine Uhr, so kann ein in A befindlicher Beobachter die Ereignisse in der unmittelbaren Umgebung von A zeitlich werten durch Aufsuchen der mit diesen Ereignissen gleichzeitigen Uhrzeigerstellungen. Befindet sich auch im Punkte B des Raumes eine Uhr -- wir wollen hinzufügen, "eine Uhr von genau derselben Beschaffenheit wie die in A befindliche" -- so ist auch eine zeitliche Wertung der Ereignisse in der unmittelbaren Umgebung von B durch einen in B befindlichen Beobachter möglich. Es ist aber ohne weitere Festsetzung nicht möglich, ein Ereignis in A mit

If at the point A of space there is a clock, an observer located at A can determine the time values of events in the immediate proximity of A by finding the positions of the hands of the clock that are simultaneous with these events. If there is at point B of space also a clock -- we want to add, "a clock with exactly the same properties as that located in A" -- it is possible for an observer at B to determine the time values of events in the immediate proximity of B. But it is not possible without further assumption to compare, in

einem Ereignis in B zeitlich zu vergleichen; wir haben bisher nur eine "A-Zeit" und eine "B-Zeit", aber keine für A und B gemeinsame "Zeit" definiert.	respect of time, an event at A with an event at B. We have so far defined only an "A-time" and a "B-time", but not time common for A and B.

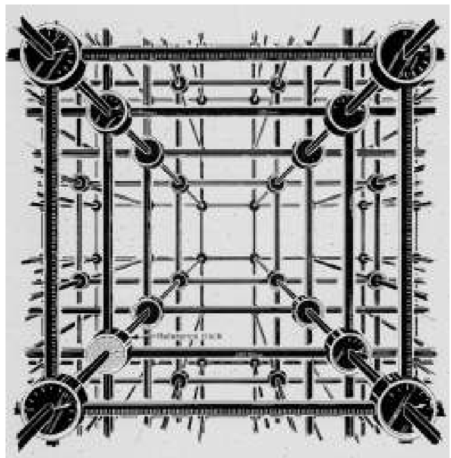
In other words we have not yet anyway to make the statement "the two events, say, two lightning strikes at two locations of the railway far away from each other, happen at the same time, i.e. simultaneously" hold any meaning. For this we need an experimental or operational procedure for measuring the simultaneity of two distant events ("Es bedarf also einer solchen Definition der Gleichzeitigkeit, daß diese Definition die Methode an die Hand gibt, nach welcher im vorliegenden Falle aus Experimenten entschieden werden kann, ob beide Blitzschläge gleichzeitig erfolgt sind oder nicht" (1916, p. 14). The method given in Einstein's 1905 (a) paper is (p. 894):

> Die [für A und B gemeinsame] Zeit kann nun definiert werden, indem man *durch Definition* festsetzt, daß die "Zeit", welche das Licht braucht, um von A nach B zu gelangen, gleich ist der "Zeit", welche es braucht, um von B nach A zu gelangen. Es gehe nämlich ein Lichtstrahl zur "A-Zeit" t_A von A nach B ab, werde zur "B-Zeit" t_B in B gegen A zu reflektiert und gelange zur "A-Zeit" t'_A nach A zurück. Die beiden Uhren laufen definitionsgemäß synchron, wenn
>
> The time [that is common to A and B] can only be defined, when one establishes by *definition* that the "time" which it takes light to reach from A to B, is the same as the "time" which it takes light to reach from B to A. Namely, let a light beam at "A-time" t_A go from A toward B, then be reflected at "B-time" t_B in B back toward A, and arrive at A again at "A-time" t'_A. The two clocks according to this definition run synchronically, when

$$t_B - t_A = t'_A - t_B$$

In his 1916 account, Einstein furnishes an even more precise definition of the simultaneity of two distant events (say, the two lightning strikes on the railway track): "The connecting line AB should be measured up along the rails, and an observer placed at the mid-point M of the distance AB. This observer should be supplied with an arrangement (e.g. two mirrors inclined at 90°) which allows him visually to observe both places A and B at the same time. If the observer perceives the two flashes of lightning at the same time, then they are simultaneous." ("Die Verbindungsstrecke AB werde dem Geleise nach ausgemessen und in die Mitte M der Strecke ein Beobachter gestellt, der mit einer Einrichtung versehen ist (etwa zwei um 90° gegeneinander geneigte Spiegel), die ihm eine gleichzeitige optische Fixierung beider Orte A und B erlaubt. Nimmt dieser die beiden Blitzschläge gleichzeitig wahr, so sind sie gleichzeitig"; 1916, p. 15.) The underlying presupposition for both of these operational definitions of "time" and "simultaneity" is that light propagates at the same speed from A to B as from B to A in the first example, or from A to M as from B to M in the second example: the speed of light in vacuum is a universal constant c. "One is thus led also to a definition of 'time' in physics. Namely one supposes that clocks of identical construction are placed at the points A, B and C of the railway line (co-ordinate system) and that they are set in such a manner that the positions of their pointers are simultaneously (in the above sense) the same. Then one understands by the 'time' of an event the reading (position of the hands) of that one of these clocks which is in the immediate vicinity (in space) of the event." ("Damit gelangt man auch zu einer Definition der 'Zeit' in der Physik. Man denke sich nämlich in den Punkten A, B, C des Geleises (Koordinatensystems) Uhren von gleicher Beschaffenheit aufgestellt und derart gerichtet, daß deren Zeigerstellungen gleichzeitig (im obigen Sinne) dieselben sind. Dann versteht man under der 'Zeit' eines Ereignisses die Zeitangabe (Zeigerstellung) derjenigen dieser Uhren, welche dem Ereignis (räumlich) unmittelbar benachbart ist"; 1916, p. 16.)

When "time" becomes so operationally precisely defined, i.e. with the synchronicity between clocks on the unit-points of the coordinate system established, one can then refine the three-dimensional Cartesian coordinate system of classical mechanics as shown below and determine the simultaneity of events on different places of this coordinate system (and so measure the spatial magnitude of any body):[2]



(adopted from J. L. Safko)

where clocks installed on the unit-points of a coordinate system (inertial frame) are synchronized. Each of the previous two inertial frames in relative motion that were posited to illustrate Galilean transformation is then to be refined in this fashion. Now a simple thought experiment immediately reveals that events that are simultaneous (having the same time coordinate) in one such coordinate system or inertial frame ("perspective") are not so in another, when the two perspectives are in (uniform translation) motion relative to one another. This invalidation of the "absolute time" of classical mechanics also explains why in the previous example the length of the rod as measured in the stationary coordinate frame will not be the same as its length as measured in the moving frame (1905a: p. 896 - 7):

Wir denken uns ferner an den beiden Stabenden (A und B) Uhren angebracht, welche mit den Uhren des ruhenden Systems synchron sind, d. h. deren Angaben jeweilen der "Zeit des ruhenden Systems" an den Orten, an welchen sie sich gerade befinden, entsprechen; diese Uhren sind also "synchron im ruhenden System".

Wir denken uns ferner, daß sich bei jeder Uhr ein mit ihr bewegter Beobachter befinde, und daß diese Beobachter auf die beiden Uhren das [vor]gestellte Kriterium für den synchronen Gang zweier Uhren anwenden. Zur Zeit ["Zeit des

We imagine further that at the two ends A and B of the rod, clocks are placed which synchronize with the clocks of the stationary system, i.e. their indications correspond at any instant to the "time of the stationary system" at the places where they happen to be. These clocks are therefore "synchronous in the stationary system."

We imagine further that with each clock there is a observer moving with it, and that these observers apply to both clocks the criterion established ruhenden Systems" und zugleich "Zeigerstellung der bewegter Uhr, welche sich an dem Orte, von dem die Rede ist, befindet"] t_A gehe ein Lichtstrahl von A aus, werde zur Zeit t_B in B reflektiert und gelange zur Zeit t'_A nach A zurück. Unter Berücksichtigung des Prinzipes von Konstanz der Lichtgeschwindigkeit finden wir:

[previously] for the synchronization of two clocks. Let a ray of light depart from A at time t_A ["the time of the stationary system," and also "the position of hands of the moving clock which is found at the place under discussion"], be reflected at B at time t_B, and reach A again at time t'_A. With consideration of the principle of the constancy of the velocity of light we find that

$$t_B - t_A = \frac{r_{AB}}{c - v}$$

und and

$$t'_A - t_B = \frac{r_{AB}}{c + v}$$

wobei r_{AB} die Länge des bewegten Stabes -- im ruhenden System gemessen -- bedeutet. Mit dem bewegten Stabe bewegte Beobachter würden also die beiden Uhren nicht synchron gehend finden, während im ruhenden System befindliche Beobachter die Uhren als synchron laufend erklären würden (p. 896 - 7).

where r_{AB} means the length of the moving rod -- measured in the stationary system. The observer moving with the moving rod would thus find the two clocks not running synchronously, while the observer located in the stationary system would declare the clocks to be synchronous.

In other words, ($t_B - t_A$) is a quantity larger than ($t'_A - t_B$) because, *from the perspective of the observer at rest relative to the moving rod*, as the rod AB moves forward so that the destination of light beam keeps moving forward, this light has to travel a larger distance during ($t_B - t_A$), while, since the speed of light is the same for both observers (moving and at rest), the observer moving with it would find ($t_B - t_A$) to be the same quantity as ($t'_A - t_B$). See figure below. For Lorentz, in this situation, the "local time" of the moving rod (e.g. $t_B - t_A$) has simply "dilated" due to its travel, while the "objective" or "real" time is the time experienced by the observer at rest. This is not the concession that Einstein makes.

```
|==========|
A          B

      tA ------>tB  (c)
      |==========|  ---->(v)
      A          B

        (c)  t'A<------tB
             |==========|  ---->(v)
             A          B
```

> Wir sehen also, daß wir dem Begriffe der Gleichzeitigkeit keine *absolute* Bedeutung beimessen dürfen, sondern daß zwei Ereignisse, welche, von einem Koordinatensystem aus betrachtet, gleichzeitig sind, von einem relativ zu diesem System bewegten System aus betrachtet, nicht mehr als gleichzeitige Ereignisse aufzufassen sind. (1905a, p. 897)
>
> We thus see that we should not attach any *absolute* meaning to the concept of simultaneity, but that two events, which are simultaneous considered from one coordinate system, can no longer be grasped as simultaneous events when considered from a moving system relative to the first.

Einstein's 1916 example, where the observer by the rail track embankment and the observer on the train moving toward the right are both timing the lightning strikes at A and B with the mirror at the mid-point M, is clearer:

> Wenn wir sagen, daß die Blitzschläge A und B in Bezug auf den Bahndamm gleichzeitig sind, so bedeutet dies: die von den Blitzorten A und B ausgehenden Lichtstrahlen begegnen sich in dem Mittelpunkte M der Fahrdammstrecke A - B. Den Ereignissen A und B entsprechen aber auch Stellen A und B auf dem Zuge. Es sei M' der Mittelpunkt der Strecke A - B des fahrenden Zuges. Dieser Punkt M' fällt zwar im Augenblick der Blitzschläge [vom Fahrdamm aus beurteilt] mit dem Punkte M zusammen, bewegt sich aber in der Zeichnung mit der Geschwindigkeit v des Zuges nach rechts. Würde ein bei M' im Zuge sitzender Beobachter diese Geschwindigkeit nicht besitzen, so würde er dauernd in M bleiben, und es würden ihn dann die von den Blitzschlägen A und B ausgehenden Lichtstrahlen gleichzeitig erreichen, d. h. diese beiden Strahlen würden sich gerade bei ihm begegnen. In Wahrheit aber eilt er [vom Bahndamm aus beurteilt) dem von B herkommenden Lichtstrahl entgegen, während er dem von A herkommenden Lichtstrahl vorauseilt. Der Beobachter wird also den von B ausgehenden Lichtstrahl früher sehen, als den von A ausgehenden. Die Beobachter, welche den Eisenbahnzug als Bezugskörper benutzen, müssen also zu dem Ergebnis kommen, der Blitzschlag B habe früher stattgefunden als der Blitzschlag A. (1916, p. 17 - 18)
>
> When we say that the lightning strokes A and B are simultaneous with respect to be embankment, we mean: the light rays emitted at the places A and B, where the lightning occurs, meet each other at the mid-point M of the length of railway embankment A - B. But the places A and B on the train also correspond to the events A and B. Let M' be the mid-point of the length A - B on the travelling train. Just when the flashes (as judged from the embankment) of lightning occur, this point M' naturally coincides with the point M but it moves towards the right in the diagram with the velocity v of the train. If an observer sitting in the position M' in the train did not possess this velocity, then he would remain permanently at M, and the light rays emitted by the flashes of lightning A and B would reach him simultaneously, i.e. they would meet just where he is situated. But in reality (considered from the railway embankment) he is hastening towards the beam of light coming from B, whilst he is riding on ahead of the beam of light coming from A. Hence the observer will see the beam of light emitted from B earlier than he will see that emitted from A. Observers who take the railway train as their reference-body must therefore come to the conclusion that the lightning flash B took place earlier than the lightning flash A. (RSGT, p. 30)

M'

```
    |---------|---------| ---> (v) train
----|---------|---------|--------- embankment
    A         M         B


       |---------|---------|  --->
----|---------|---------|---------
    @----->   |  |<-----@  (c) light signals from lightning strokes
    A         M  M'     B
```

While for the observer at the embankment (the observer at rest), the lightning strokes are simultaneous because AM/c = MB/c (since AM = MB), for the observer on the train (the moving observer) they are not simultaneous because AM/(c - v) > BM/(c + v).

Now Einstein's concern is, if, assuming the compatibility between the constancy of the speed of light in vacuum (irrespective of the velocity of its source) and the principle of relativity, *thus foregoing the simultaneity valid in all possible inertial frames*, one tries to find the equations of transformation which, like the Galilean transformation, accord with the principle of relativity, but which, unlike Galilean transformation, preserve the constancy of the speed of light in the two inertial frames, what those equations will be. As Einstein reflects in 1916:

> Gibt es eine solche denkbare Antwort auf diese Frage, daß das Gesetz der Lichtausbreitung im Vakuum dem Relativitätsprinzip nicht widerspricht? Anders ausgedrückt: Ist eine Relation zwischen Ort und Zeit der einzelnen Ereignisse im bezug auf beide Bezugskörper denkbar, derart, daß jeder Lichtstrahl relativ zum Bahndamm und relativ zum Zug die Ausbreitungsgeschwindigkeit c besitz?... (1916, p. 20 - 1)
>
> Is there a thinkable answer to this question of such a nature that the law of transmission of light in vacuo does not contradict the principle of relativity? In other words: Can we conceive of a relation between place and time of the individual events relative to both reference-bodies, such that every ray of light possesses the velocity of transmission c relative to the embankment and relative to the train?... (RSGT, p. 35).
>
> Die Beziehungen müssen so gewählt werden, daß dem Gesetz der Vakuumfortpflanzung des Lichtes für einen und denselben Lichtstrahl (und zwar für jeden) in bezug auf K und K' Genüge geleistet wird. (1916, p. 21 - 2)
>
> The relations must be so chosen that the law of the transmission of light in vacuo is satisfied for one and the same ray of light (and of course for every ray) with respect to K and K' (RSGT, p. 36 - 7).

So, taking light itself as the event, through an *algebraic* analysis of the relationship between the coordinates in the two systems, (x, y, z, t) and (x', y', z', t'), of a light signal emitted and reflected back in the above manner, and which preserves the same velocity in both systems, Einstein in the 1905 (a) paper ("3. Theorie der Koordinaten- und Zeittransformation von dem ruhenden auf ein relativ zu diesem in gleichförmiger Translationsbewegung befindliches System") deduces *de novo* the transformation equations that govern (serves as the *eidoi* for) this relationship, which has to be a *linear* relationship, between the light's (x, y, z, t) coordinates and its (x', y', z', t') coordinates. Those transformation equations, of course, are just those of Lorentz Transformation. The demonstration that the Lorentz transformation represents just the equations which accord with the principle of relativity and preserve at the same time the constancy of the speed of light in both inertial frames, all the while renouncing absolute simultaneity -- or rather, the "objective, real" time of the rest frame (since it is no longer absolutely at rest, but only so relative to... in accordance with the principle of relativity) -- shows therefore that the *eidos* that Lorentz has discovered really points to a different physical make-up of the universe than does his modification of the shadows that we have been used to (the "objective" space and time of classical mechanics modified with the contraction of objects by ether). All spatial distances and all time intervals are "local", and not any of them can claim themselves to be more "real" or "objective" than the others. The combination of the constancy of speed of light irrespective of the velocity of its source with Galilean relativity -- which Lorentz excludes -- means not only that this constancy of the speed of light is not the function of some "ether" absolutely at rest, but of Galilean relativity -- hence light's velocity stays constant no matter from which direction one attempts to measure it (irrespective of the direction and velocity of the observer), and there is no contradiction between Fizeau's experiment and Michelson and Morley's -- but also "the dethronment of Newtonian absolute time, common to the whole universe, and its replacement by a multiplicity of individual times, discordant among themselves" (TD, p. 34). Hence, even though, as Damour has noted, most of the mathematical equations have already been found, before or at the same time, by Lorentz and Poincaré (TD, p. 27), "the essential point is that the physical meaning of these equations was completely new with Einstein" (p. 30; "Le point essentiel est que le sens physique de ces équations était complètement nouveau chez Einstein"), and it is this new physical meaning which constitutes the "step" (den Schritt) which Einstein said (in a conversation with Abraham Pais) special relativity represented: in Damour's words, a "major conceptual discontinuity" (*discontinuité conceptuelle majeure*; p. 27). In Kuhn's terminology, Lorentz and Poincaré did not constitute a "scientific revolution" because they worked essentially within the existing Newtonian paradigm, making some local modifications therein (adding "local times"), while in special relativity familiar things (e.g. spatial distances and time intervals) were looked at differently: one saw new things while looking at old things: a new paradigm, a scientific revolution. In fact, Lorentz himself later on would acknowledge that this failure to see a different world while the opportunity was there (his transformation equations) was precisely his mistake.[3]

In the next section of the 1905 (a) paper (4. Physikalische Bedeutung der erhaltenen Gleichungen, bewegte starre Körper und bewegte Uhren betreffend, "The physical meaning of the equations obtained regarding moving rigid bodies and moving clocks"), Einstein explores the practical (experimental) consequences of this new relativity of space and time (all spatial and temporal measurements being local only) by demonstrating that a sphere ("that is, a body which when examined while at rest possesses a spherical shape"; "Das heißt einen Körper, welcher ruhend untersucht Kugelgestalt besitzt"; p. 903) which relative to the system in which it is at rest (and in whose beginning (0, 0, 0) lies its center) has its surface expressed by $\xi^2 + \eta^2 + \zeta^2 = R^2$, when measured from the system in uniform translation movement relative to it, has its surface expressed by (at time t = 0 when the beginnings of the two systems coincide):

$$x^2 / [\sqrt{1 - (v/c)^2}]^2 + y^2 + z^2 = R^2$$

> Ein starrer Körper, welcher in ruhendem Zustande ausgemessen die Gestalt einer Kugel hat, hat also in bewegtem Zustande -- vom ruhenden System aus betrachtet -- die Gestalt eines Rotationsellipsoides mit den Achsen
>
> A rigid body which, measured in a state of rest, has the form of a sphere, has thus in a state of motion -- considered from the stationary system -- the form of an ellipsoid of rotation with the axes:

$$R\sqrt{1-(v/c)^2},\ R,\ R.$$

(p. 903). In the previous example of the moving rod -- contrary to the unspoken assumption of classical mechanics, when measured from the rest inertial frame its length would not be *l*. In fact, when the moving observer in (a) measures its length *l* to be 1 meter, the observer at rest would measure it to be $(1 - (v/c)^2)^{1/2}$ of a meter (1916, p. 24). Lorentz's "length-contraction of the moving body" thus is valid only for the observer who stays at rest relative to the moving body; from the perspective of the observer moving with the body such that it is at rest relative to him or her, there is no length-contraction. We therefore see it demonstrated in concrete that the difference in physical meaning between Einstein's interpretation of Lorentz transformation and Lorentz' own interpretation is not trivial but has major experimental consequences, caused by a difference in the physical make-up of the universe.[4] Similarly with time:

Wir denken uns ferner eine der Uhren, welche relativ zum ruhenden System ruhend die Zeit t, relativ zum bewegten System ruhend die Zeit τ anzugeben befähigt sind, im Koordinatenursprung von k gelegen und so gerichtet, daß sie die Zeit τ angibt. Wie schnell geht diese Uhr, vom ruhenden System aus betrachtet?	We further imagine one of the clocks which are qualified to give the time t when at rest relative to the stationary system, and the time τ when at rest relative to the moving system, to be located at the coordinate origin of k, and so adjusted, that it gives the time τ. How fast will this clock run, when considered from the stationary system?
Zwischen die Größen x, t und τ, welche sich auf den Ort dieser Uhr beziehen, gelten offenbar die Gleichungen:	Between the quantities x, t, and τ, which refer to the position of the clock, the equations evidently are valid:

$$\tau = 1/[\sqrt{1-(v/c)^2}\,]\,[t - [1 - (v/c^2)x]$$

und and

$$x = vt$$

Es ist also Therefore

$$\tau = t\sqrt{1-(v/c)^2} = t - [1 - \sqrt{1-(v/c)^2}\,]\,t,$$

woraus folgt, daß die Angabe der Uhr (im ruhenden System betrachtet) pro Sekunde um $[1 - (1 - (v/c)^2)^{1/2}]$ Sek. oder -- bis auf Größen vierter und höherer Ordnung um $[1/2(v/c)^2]$ Sek. zurückbleibt.	whence it follows that the time given by the clock (viewed from the stationary system) is behind by $[1 - (1 - (v/c)^2)^{1/2}]$ per second, or -- [neglecting] magnitudes of the fourth or higher order -- by $[1/2(v/c)^2]$.

Hieraus ergibt sich folgende eigentümliche Konsequenz. Sind in den Punkten A und B von K ruhende, im ruhenden System betrachtet, synchron gehende Uhren vorhanden, und bewegt man die Uhr in A mit der Geschwindigkeit v auf der Verbindungslinie nach B, so gehen nach Ankunft dieser Uhr in B die beiden Uhren nicht meht synchron, sondern die von A nach B bewegte Uhr geht gegenüber der von Anfang an in B befindlichen um [$1/2tv^2/c^2$] Sek. (bis auf Größen vierter und höheren Ordnung) nach, wenn t die Zeit ist, welche die Uhr von A nach B braucht.

From this ensues the following peculiar consequence. If at the points A and B of K there are stationary clocks, which, viewed in the stationary system, are synchronous; and if one moves the clock at A with velocity v along the connection line to B, then on its arrival at B the two clocks are no longer running synchronously, but the clock moved from A to B lags behind the clock located at B since the beginning by [$1/2tv^2/c^2$] (up to magnitudes of fourth and higher order), where t is the time which the clock needs to go from A to B.

With the "re-interpretation" of the Lorentz Transformation, the very meaning of the coordinate system (the quantitative representation of space and time) and its coordinates has changed.

> Daß wir aus den Transformationsgleichungen etwas über das physikalische Verhalten von Maßstäben und Uhren erfahren müssen, liegt a priori auf der Hand. Denn die Größen x, y, z, t sind ja nicths anderes als mit Maßstäben und Uhren zu gewinnende Meßresultate. (1916, p. 25)
>
> That we must learn something about the physical behavior of measuring-rods and clocks from the equations of transformation, is a priori quite clear. For the magnitudes z, y, x, t are nothing other than the results of measurements obtained with measuring-rods and clocks..

The spatial and temporal measurements are no longer objective representation of space and time independent of the experimenter, but are space-and-time itself, which is dependent on the condition of motion of the experimenter.

Einstein's 1905 (a) classics is divided into the two sections on kinematics (dealing with rigid bodies) and electrodynamics (dealing with waves). The above has laid the foundation for a fundamental revision of the Newtonian kinematics (relativistic electrodynamics will not be considered here). Under this theme, the Newtonian laws of motion now all turn out to be approximations, i.e. shadows, of the real laws, the real "forms". First, the Galilean transformation itself is the shadowy approximation of Lorentz transformation, in the sense that it can be derived from the latter if the value of the speed of light is posited as infinite (1916, p. 23).[5] That is, for our everyday eye, in our less precise, illusory everyday perception, light *seems* to travel infinitely fast, it *approximately* has an infinite velocity, hence when we quantify it in our "approximate" everyday framework with diminished visibility (in Plato's way of speaking), we get Galilean transformation. Galilean transformation is an "image" of Lorentz transformation, to speak in Platonic terms. Grasping the precise velocity of light, the Lorentz transformation then gives the new, *real* theorem of the addition of velocities of which the old theorem is simply a shadowy distortion, an "image reflection" (as on water or through the air). If, for example, a point moves along the x' axis of frame k' with a velocity w, its velocity U from the point of view k is not w + v, but:

$$U = \frac{v + w}{1 + (vw/c^2)}$$

Aus dieser Gleichung folgt, daß aus der Zusammensetzung zweier Geschwindigkeiten, welche kleiner sind als [c], stets eine Geschwindigheit kleiner als [c] resultiert... Es folgt ferner, daß die Lichtgeschwindigkeit [c] durch Zusammensetzung mit einer "Unterlichtgeschwindigkeit" nicht geändert werden kann.

It follows from this equation that, from the combination of two velocities which are less than [c], a velocity less than [c] always results.... It follows further, that the velocity of light [c] cannot be altered by combination with another velocity less than the velocity of light.

This second condition explains why light's velocity is always the same, independent both of the velocity and direction of its source (Fizeau's experiment) *and* of the velocity and direction of the observer (Michelson-Morley's experiment). This new theorem of the addition of velocities is really the result of the principle of relativity, being valid in all inertial frames in uniform translation movement relative to one another (the point's velocity measured in frame k', for example, is really $w = (0 + w)/(1 + 0/c^2) = w/1 = w$, since $v = 0$), thus both the Fizeau experiment and the Michelson-Morley experiment are explained by the principle of relativity, and not by the stationary ether in one and the contraction of length in the other.

Then (1905a: "10. Dynamik des (langsam beschleunigten) Elektrons", "the dynamics of the slowly accelerated electron"; see also 1910, p. 142 - 2) when Einstein applies the relativistic transformation to the motion of the electron at lower velocity where classical kinematics still holds for it (with only the addition of charge), he demonstrates that the old Newtonian *eidos* for force $F_x = d/dt(m(dx/dt))$ (and similarly for the y and z axes) is just a shadowy approximation that will work only in the rest frame but not in the one in uniform translation movement relative to it. It is not the *eidos* that will be compatible with the principle of relativity. The *real form* that would be valid in all inertial frames -- of which the old form is the less precise shadow -- is:

$$F_x = \frac{d}{dt}\left[\left(m\frac{dx}{dt}\right) / \sqrt{1 - (v^2/c^2)}\,\right]$$

(and similarly for the y and z axes). In the frame taken as rest (i.e. which moves with the mass being measured) or when the velocity of the mass being measured (with respect to the inertial frame, of course) is extremely small compared with c, the relativistic (or Lorentz) factor $(1 - v^2/c^2)^{1/2}$ is or is nearly 1 ($((1 - 0^2/c^2)^{1/2} = (1)^{1/2} = 1$; same as when c is infinite), so that d/dt(mv) holds approximately.

With this, the kinetic energy *eidos* $E = 1/2mv^2$ is not the real *eidos* but only the distorted, approximate form or "image reflection" of the latter, which is:

$$E = \frac{m_0 c^2}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Again, when the velocity of the body measured (relative to the frame of reference) is small as compared with the speed of light and the Lorentz factor is about 1, since kinetic energy is the difference between the total energy of the body while in motion and its rest energy:

$$m_0c^2(\frac{1}{\sqrt{1-\frac{v^2}{c^2}}}-1)$$

the difference approximates to $1/2mv^2$. On the other hand,[6] the real expression of kinetic energy

> ... wird unendlich, wenn sich die Geschwindigkeit v der Lichtgeschwindigkeit c nähert. Es muß also die Geschwindigkeit stets kleiner als c bleiben, wie große Energien man auch auf die Beschleunigung verwenden mag. (1916, p. 30)
>
> ... becomes infinite, when the velocity v [of the mass relative to the inertial frame from which kinetic energy is measured] approaches the velocity of light c [mc^2 divided by the square root of 0] . The velocity [of any mass measured] must therefore remain less than c, however great the energy one may apply [to produce] the acceleration.

Thus, the fact that no object that isn't already traveling at c can travel at c can be derived: both the kinetic energy and the force needed to produce the velocity, and the inertia (m) of its mass, all increase toward infinity as its velocity approaches c, and its time nearly stops (to the rest frame). In the case of the spherical body earlier, at the speed of light it would shrivel up into a plane figure (in the direction of its motion). "For velocity greater than light our considerations become senseless" (1905a, p. 903) because the square root in the Lorentz factor would become imaginary. It is in this sense that velocity c represents the "unattainable limiting speed" (die unerreichbaren Grenzgeschwindigkeit; 1916, p. 24 - 5).

The relativity of space and time measurements as the reflection of the conservation of distance in the four-dimensional space-time continuum. Space and time in special relativity has been unified into a single entity of a four dimensional space-time continuum, with time as the fourth dimension added to the three dimensional space, because (the new interpretation of) Lorentz transformation has demonstrated that the time coordinate of an event is not independent of, but tied to, the spatial coordinates. Here another important figure, Hermann Minkowski, comes onto the scene. The four dimensional space-time continuum is analogous to the three dimensional continuum of space:

Der Raum ist ein dreidimensionales Kontinuum. Dies will sagen, daß es möglich ist, die Lage eines (ruhenden) Punktes durch drei Zahlen (koordinaten), x, y, z, zu beschreiben, und daß es zu jedem Punkte beliebig "benachbarte" Punkte gibt, deren Lage durch solche Koordinatenwerte (Koordinaten) x_1, y_1, z_1 beschrieben werden kann, die den Koordinaten x, y, z des erstgenannten beliebig nahe kommen. Wegen der letzteren	Space is a three-dimensional continuum. This means that it is possible to describe the position of a point (at rest) by means of three numbers (co-ordinates) x, y, z, and that, to it, there is an indefinite number of "neighboring" points, the position of which can be described by coordinate values (co-ordinates) such as x_1, y_1, z_1, which may be as near as we choose to the respective values of the co-ordinates x, y, z, of the first point.

Eigenschaft sprechen wir von "Kontinuum" wegen der Dreizahl der Koordinaten von "dreidimensional".

Analog ist die Welt des physikalischen Geschehens, von Minkowski kurz "Welt" genannt, natürlich vierdimensional in zeitraümlichen Sinne. Denn sie setzt sich aus Einzelereignissen zusammen, deren jedes durch vier Zahlen, nämlich drei räumliche Koordinaten x, y, z und eine zeitliche Koordinate, den Zeitwert t beschrieben ist. Die "Welt" ist in diesem Sinne auch ein Kontinuum; denn es gibt zu jedem Ereignis beliebig "benachbarte"... Ereignisse, deren Koordinaten x_1, y_1, z_1, t_1 sich von denen des ursprünglich betrachteten Ereignisses x, y, z, t beliebig wenig unterscheiden. Daß wir nicht daran gewöhnt sind, die Welt in diesem Sinne als vierdimensionales Kontinuum aufzufassen, liegt darin, daß die Zeit in der vorrelativistischen Physik gegenüber den räumlichen Koordinaten eine verschiedene, mehr selbständige Rolle spielt. Darum haben wir uns daran gewöhnt, die Zeit als ein selbständiges Kontinuum zu behandeln. In der Tat ist die Zeit gemäß der klassischen Physik absolut, d. h. vom der Lage und dem Bewegungszustande des Bezugssystems unabhängig. Dies kommt in der letzten Gleichung der Galilei-Transformation (t' = t) zum Ausdruck.

Durch die Relativitätstheorie ist die vierdimensionale Betrachtungsweise der "Welt" geboten, da ja gemäß dieser Theorie die Zeit ihrer Selbständigkeit beraubt wird, wie die vierte der Gleichungen der Lorentz-Transformation $t' = (t - (v/c^2)x)/(1- v^2/c^2)^{1/2}$ lehrt. Denn nach dieser Gleichung verschwindet die Zeitdifferenz Δt'

In virtue of the latter property we speak of a "continuum," and owing to the fact that there are three co-ordinates we speak of it as being " three-dimensional."

Similarly, the world of physical happenings, which was briefly named "world" by Minkowski, is naturally four dimensional in the space-time sense. For it is composed of individual events, each of which is described by four numbers, namely, three space co-ordinates x, y, z, and a time co-ordinate, the time value t. The" world" is in this sense also a continuum; for to every event there are as many "neighbouring" events... as we care to choose, whose co-ordinates x_1, y_1 z_1, t_1 differ by an indefinitely small amount from those of the event x, y, z, t originally considered. That we have not been accustomed to regard the world in this sense as a four-dimensional continuum is due to the fact that in the pre-relativity physics, time played a different and more independent role, as compared with the space coordinates. It is for this reason that we have been in the habit of treating time as an independent continuum. In fact, according to classical mechanics, time is absolute, i.e. it is independent of the position and the condition of motion of the coordinate system. This comes to expression in the last equation of the Galilean transformation (t' = t).

Through the theory of relativity the four-dimensional mode of consideration of the "world" is demanded, since according to this theory time is robbed of its independence, as shown by the fourth equation of the Lorentz transformation: $t' = (t - (v/c^2)x)/ (1- v^2/c^2)^{1/2}$ For, according to this equation, the time difference Δt' of two events with respect to K' does not in general vanish, even

zweier Ereignisse in bezug auf K' auch dann im allgemeinen nicht, wenn die Zeitdifferenz Δt derselben in bezug auf K verschwindet. Rein räumliche Distanz zweier Ereignisse in bezug auf K hat zeitliche Distanz derselben in bezug auf K' zur Folge.	when the time difference Δt of the same events with reference to K vanishes. Pure "space-distance" of two events with respect to K results in "time-distance" of the same events with respect to K'.

But the real importance of Minkowski's four-dimensional "world" lies in its formalism that establishes a clear relationship with the three-dimensional Euclidean space. When Minkowski first announced this formalism for the relativity theory in 1908 (Sept. 21, at the 80th Congress of German Scientists and Doctors at Cologne [Gesellschaft der Arzte und Naturforscher], TD, p. 51; the presentation later appeared as "Raum und Zeit" in *Physikalische Zeitschrift*, 10 p. 104 - 111; translated as "Space and Time" in *Principle of Relativity*, p. 75), Einstein was not at ease with it. Later he accepted it whole-heartedly. It starts with a simpler derivation of the Lorentz Transformation (1916, p. 81).

A light-signal sent out from the origin of K at the time t = 0 will travel a distance r:

$$r = \sqrt{x^2 + y^2 + z^2} = ct$$

which equation after squaring becomes:

$$x^2 + y^2 + z^2 = c^2t^2$$

which becomes:

$$x^2 + y^2 + z^2 - c^2t^2 = 0$$

The constancy of the speed of light in all inertial coordinate systems or perspectives means ("Das Gesetz von der Lichtausbreitung in Verbindung mit dem Relativitätspostulat verlangt, daß die Ausbreitung des nämlichen Signals -- Von K' aus beurteilt -- nach der entsprechenden Formel [shown below] erfolge.")

$$r' = ct'$$

or

$$x'^2 + y'^2 + z'^2 - c^2t'^2 = 0$$

Since the constancy of the speed of light means that, for a light signal proceeding along the positive axis x according to the equation x = ct or x - ct = 0 in K, or along (positive) x' according to x' - ct' = 0 in K', the Lorentz Transformation must satisfy the condition

$$x' - ct' = x - ct$$

or

$$x'^2 - c^2t'^2 = x^2 - c^2t^2$$

the following must also hold with Lorentz Transformation:

$$x'^2 + y'^2 + z'^2 - c^2t'^2 = x^2 + y^2 + z^2 - c^2t^2$$

That is, as Einstein wrote in an footnote added in 1913 to his 1905 (a) paper: "Die Gleichungen der Lorentz-Transformation sind einfacher direkt aus der Bedingung abzuleiten, daß vermöge jener Gleichungen die Beziehung $\xi^2 + \eta^2 + \zeta^2 - c^2\tau^2 = 0$ die andere $x^2 + y^2 + z^2 - c^2t^2 = 0$ zur Folge haben soll." ("The equations of Lorentz transformation may be more simply derived directly from the condition that, in virtue of these equations, the relation $\xi^2 + \eta^2 + \zeta^2 - c^2\tau^2 = 0$ should have as its consequence the other one $x^2 + y^2 + z^2 - c^2t^2 = 0$.") This is when the Lorentz Transformation has gone through a "generalization" (Verallgemeinerung). "Es ist offenbar unwesentlich, daß die Achsen von K' denen von K räumlich parallel gewählt werden. Es ist auch unwesentlich, daß die Translationsgeschwindigkeit von K' gegenüber K die Richtung der x-Achse hat. Man kann die Lorentz-Transformation in diesem allgemeinen Sinne -- wie eine einfache Überlegung ergibt -- zusammensetzen aus zweierlei Transformationen, nämlich aus Lorentz-Transformationen im speziellen Sinne und aus rein räumlichen Transformationen, welche der Ersetzung des rechtwinkligen Koordinatensystems durch ein neues mit anders gerichteten Achsen entspricht." (1916 p. 82; "Obviously it is inessential whether the axes of K' be chosen so as to be spatially parallel to those of K. It is also inessential that the translation velocity of K' with respect to K should be in the direction of the x-axis. As a simple consideration shows, one can construct the Lorentz transformation in this general sense from two kinds of transformations, viz. from Lorentz transformations in the special sense and from purely spatial transformations, which correspond to the replacement of the rectangular co-ordinate system by a new system with its axes pointing in other directions.") In other words, the Lorentz transformation actually expresses some sort of "conservation" as one moves from one perspective or coordinate system to another that is in uniform translation movement relative to the first one. We'll see presently what it is that is being conserved here.

Now, in the Minkowski four dimensional space time, as Poincaré has first discovered (see note 3), and then as Minkowski has elaborated, the only way to measure the separations between events that is consistent with Lorentz Transformation is by the rule[7]: $ds^2 = dx^2 + dy^2 + dz^2 - c^2dt^2$. As Minkowski discovers, the Lorentz transformation requires (or satisfies the condition) that the above relationship derived for the coordinates themselves of a single event in two coordinate systems in uniform translation be also valid for the coordinate differences between two events in two such coordinate systems, for the differentials of these coordinate differences (1916, p. 62):

$$dx^2 + dy^2 + dz^2 - c^2dt^2 = dx'^2 + dy'^2 + dz'^2 - c^2dt'^2$$

"This condition has as a consequence the validity of the Lorentz transformation. We can express this as follows: The magnitude which belongs to two adjacent points of the four-dimensional space-time continuum: ("Diese Bedignung hat die Gültigkeit der Lorentz-Transformation zur Konsequenz. Wir können das so aussprechen: Die zu zwei benachbarten Punkten des vierdimensionalen raum-zeitlichen Kontinuums gehörige Größe:)

$$ds^2 = dx^2 + dy^2 + dz^2 - c^2dt^2$$

has the same value for all selected (Galileian) reference-bodies" (hat für alle bevorzugten (Galileischen) Bezugskörper denselben Wert." (Ibid.) The technical way of expressing this equation is to say that the distance between two events in the four dimensional space-time is "invariant with respect to Lorentz Transformation." ("Mathematisch entspricht dieser Tatsache der Umstand, daß ds^2 in bezug auf Lorentz-Transformationen invariant ist." 1916, p. 99) That is, the space-time distance between two events is invariant or "conserved" as one moves from one perspective or coordinate system to another that is in uniform translation movement relative to the first one: the invariance of space-time distance or the conservation of space-time difference. The spatial contraction and time dilation that are experienced between two such coordinate systems, together with the law of the constancy of the speed of light in vacuum, or, in another manner, the limitation of possible velocities to no more than c, are all *effects* of the invariance or conservation of space-time distance despite relative motion of the measuring inertial frames (*structure*), such that the components that make up this four dimensional distance have to change from one inertial frame to another in order to conserve the total amount. The "simpler derivation" or "generalization" of Lorentz Transformation is the demonstration of this, and Lorentz Transformation itself is the mechanism allowing the general conservation of space-time to show up in the specific local effects of the measurement of space and time in one particular inertial frame. In other words, formerly, when consciousness was, in its activities of measuring space and time (motion), limited without knowing to a single inertial frame, it thought that there was on the one hand the conservation or invariance of space distance, and on the other, the conservation or invariance of time interval: the Newtonian absolute space and absolute time, or the three-plus-one dimension.[8] Now, as it has enlarged its horizon, and has learned (during measurement of high-velocity motion) to take account of two inertial frames at once, it realizes that the universe is four-dimensional (not three-plus-one), so that it is the space-time interval which will be invariant or conserved, not the spatial distance and the time interval separately. When measuring a particular space-time interval, not a space interval and a time interval, all will get the same value, no matter how fast they are travelling with respect to one another. In this connection, the law of the propagation of light can be derived in two ways. The first way is to see c as a conversion factor that converts the usual units of time to the same units used for space. That is, cdt measures the time interval in units of length. (Or, reversely, the use of "light-year" or "light-second," i.e. 300,000 km, converts distance to time. TD, p. 60.) If c is a conversion factor, it is then no surprise, not simply posited so, that it is the same to all observers in (uniform translation) motion relative to one another. (JLS) The second way is to notice that, when a body is at rest (in the coordinate system that moves with it), between event 1 (the body at t = 0) and event 2 (the body at t = 1), there is no space displacement ($dx_1^2 + dx_2^2 + dx_3^2 = 0$); there is only time displacement; that, on the other hand, the interval between two events (two points in the "fieldfree" Minkowski four dimensional space-time) connected by light (e.g. the light signal at t = 0 as event 1, and at t = 1 as event 2) is zero, since, as seen above, the displacement in space, dx, is equal to the time displacement, cdt ($x^2 + y^2 + z^2 - c^2t^2 = 0$). (This not only explains why [to the observer at rest] time stops for the object that reaches velocity c, but also means that nothing can go faster than c because the square of spacetime distance along an observer's worldline can't be negative.) This space-time interval of zero only occurs when the velocity is c. This particular value of zero for the interval is unique in the sense that it is the only value of the space-time interval for which v can have one and only one value, no matter in which inertial frame it is measured. This shows that not only is the velocity of light constant, it is the only velocity constant. Therefore, the constancy of the speed of light is not a postulate because, when observers observe a zero space-time interval, they all observe it to have a velocity of c, no matter how fast they are moving relative to each other.

As an example for the above rule, the spacetime distance ds^2 between event G (say, a supernova) whose coordinates are ($ct_G = 5.0$ m, $x_G = 3.0$ m, $y_G = 2$ m, $z_G = 0.0$ m) and event H (say, another supernova) whose coordinates are ($ct_H = 6.0$ m, $x_H = 3.0$ m, $y_H = 2.5$ m, $z_H = 0.0$ m), where m = 1 million light-year, would be $(3 - 3)^2 + (2.5 - 2)^2 + (0 - 0)^2 - (6 - 5)^2 = (0)^2 + (0.5)^2 + (0)^2 - (1)^2 = 0.25 - 1 = - 0.75$ m^2 (this does not mean that the square of spacetime distance is negative, because the use of (+---) would give 0.75 m^2), so that ds = 0.87m. (JLS) Now, "when [ds^2] is negative [using (+++-)], this means that the two events can be connected by a massive object (an atom, an observer) which moves with a velocity lower than that of light. In this case, the squared interval between the two events, once its sign be changed and it be divided by the square of the velocity of light, is equal to the time lived by this atom or observer going from one event to another at a constant speed." Thus, 0.75 m^2/c^2 would be this time. Then, "in the case where the squared interval between the two events is positive, that means that there exists an observer for whom these two events are simultaneous and separated by a spatial distance whose square is equal to this squared interval." (TD, p. 59 - 60. "Quand il est négatif, cela veut dire que les deux événements peuvent être reliés par un objet massif (un atome, ou un observateur) qui se déplace à une vitesse inférieure à celle de la lumière. Dans ce cas, l'intervalle carré entre les deux événements est égal, une fois changé de signe et divisé par le carré de la vitesse de la lumière, au carré du temps vécu par cet atome ou cet observateur pour passer, à vitesse constante, d'un événement à l'autre. Enfin, dans le cas où l'intervalle carré entre deux événements est positif, cela veut dire qu'il existe un observateur pour lequel ces deux événements sont simultanés et séparés d'une distance spatiale dont le carré est égal à cet intervalle carré.")

Now the question remains: what does motion do to space-time such that the components thereof have to change in order to conserve the same total? Since in a four dimensional Euclidean space the spatial difference between two points can be derived through the expansion of the Pythagorean theorem for the two dimension into four dimension: $ds^2 = dx^2 + dy^2 + dz^2 + d\alpha^2$, Minkowski finds that if he substitutes the imaginary square root of -1 times ct (ict) for the time variable t, he can then make the four dimensional spacetime of relativity the exact same thing as the Euclidean four dimensional space. Since

$$(\sqrt{-1}\ ct)^2 = (\sqrt{-1})^2(ct)^2 = -c^2t^2$$

then,

$$ds^2 = dx^2 + dy^2 + dz^2 - c^2dt^2$$

becomes

$$ds^2 = dx^2 + dy^2 + dz^2 + d(\sqrt{-1}\ ct)^2$$

Then, with

$$x = x_1$$
$$y = x_2$$
$$z = x_3$$

$$t = (\sqrt{-1}\ ct) = x_4$$

an exact analogy is established with equations in the four dimensional Euclidean space, for both the coordinate transformation between two perspectives and the distance between two points:

$$x_1^2 + x_2^2 + x_3^2 + x_4^2 = x'^2_1 + x'^2_2 + x'^2_3 + x'^2_4$$

$$ds^2 = dx_1^2 + dx_2^2 + dx_3^2 + dx_4^2$$

Hence: "Man kann die Minkowskische Welt formal als einen vierdimensionalen euklidischen Raum (mit imaginärer Zeitkoordinate) ansehen; die Lorentz-Transformation entspricht einer 'Drehung' des Koordinatensystems in der vierdimensionalen 'Welt'." ("We can regard Minkowski's 'world' in a formal manner as a four-dimensional Euclidean space (with an imaginary time coordinate); the Lorentz transformation corresponds to a 'rotation' of the co-ordinate system in the four dimensional 'world'"; 1916, p. 83.)[9] In this way, a time-connoting "event" (event-point: *Punktereignis*) in the Minkowskian world becomes no different than a spatial point in the timeless Euclidean space, as a "world-point" (*Weltpunkt*): "Die Physik wird aus einem *Geschehen* im dreidimensionalen Raum gewissermaßen ein *Sein* in der vierdimensionalen 'Welt'" (ibid.; "From a 'happening' in three-dimensional space, physics becomes, as it were, a 'be' in the four-dimensional 'world'"). Thus, we say that reality is given *en bloc*. (The full implication of this extremely important insight will be considered below.) "Motion" then means the rotation of space-time coordinate system, and just as a length remains invariant during its rotation on a two dimensional plane, so a space-time interval will remain invariant during its rotation in the four-dimensional space-time -- and a distance of zero will always be zero (the constancy of the speed of light). This is the larger picture that consciousness sees as it gets out of its provincialness, its cave, to see what it is that is casting the shadows or images that it saw before on the wall in the cave.

Einstein summarizes the analogy in a 1921 note:

dreidimensionale euklidische Geometrie three-dimensional Euclidean geometry	spezielle Relativitätstheorie theory of special relativity
Zu zwei benachbarten Raumpunkten gehört eine Masszahl (Abstand ds), gemäß der Gleichung $ds^2 = dx_1^2 + dx_2^2 + dx_3^2$, welche unabhängig vom gewählten Koordinatensystem [definiert] ist und mit dem Einheitsmaßstab messbar ist. To two neighboring spatial points belongs a measurement (distance ds) according to the equation $ds^2 = dx_1^2 + dx_2^2 + dx_3^2$, which is [defined] independently of the chosen coordinate system and is measurable with a unit measuring rod.	Zu zwei benachbarten Raum-Zeit-Punkten (Punktereignissen) gehört eine Masszahl (Abstand ds) gemäß der Gleichung $ds^2 = dx_1^2 + dx_2^2 + dx_3^2 + dx_4^2$, welche unabhängig vom gewählten [Koordinaten]inertialsystem ist und mit dem Einheitsmaßstab und Einheitsuhr gemessen werden kann. To two neighboring space-time points (event-points) belong a measurement (distance ds) according to the equation $ds^2 = dx_1^2 + dx_2^2 + dx_3^2 + dx_4^2$, which is independent of the chosen inertial [coordinate] system and can be measured with a unit measuring rod and a unit clock.
Die zulässigen Transformationen sind dadurch charakterisiert, daß sie den Aussdruck für ds^2 zur Invarianten haben; d. h. es sind zulässig die linearen orthogonalen	Die zulässigen Transformationen sind dadurch charakterisiert, daß sie den Ausdruck für ds^2 zur Invarianten haben; d. h. es sind zulässig diejenigen linearen orthogonalen Substitutionen, welche den Realitätscharakter von x_1, x_2, x_3, x_4

Transformationen. The permissible transformations are characterized by this, that they have as invariant the expression of ds^2; i.e. the linear orthogonal transformations are permissible.	aufrecht erhalten. Es sind dies die Lorentz-Transformationen. The permissible transformations are characterized by this, that they have as invariant the expression of ds^2; i.e. those linear orthogonal substitutions, which maintain the character of x_1, x_2, x_3, x_4 as real numbers. These are the Lorentz transformations.
Diesen Transformationen gegenüber sind die Gesetze der euklidischen Geometrie invariant. With respect to these transformations the laws of Euclidean geometry are invariant.	Diesen Transformationen gegenüber sind die Gesetze der Physik invariant. With respect to these transformations the laws of physics are invariant.

The equivalence between mass and energy Three months after the foundational paper on the electrodynamics of moving bodies, Einstein discovered another implication of the new theory, a certain equivalence between inertial mass and energy, that "the principle of relativity, together with Maxwell's equations, requires that mass be a direct measure of the energy contained in a body." ("Das Relativitätsprinzip im Zusammenhang mit den Maxwellschen Grundgleichungen verlangt nämlich, daß die Masse direkt ein Maß für die im Körper enthaltene Energie ist." Einstein's letter to Corad Habicht, quoted in CP, TR, p. 268 - 9) Already, the aforementioned *real eidos* for kinetic energy shows that, if the body be at rest, then the Lorentz Factor would be 1, and the rest energy of the body -- its inertial mass -- would have the value of mc^2. In the first paper on the subject (1905 b), Einstein was only able to establish the mass-energy equivalence for the kind of energy possessed by electromagnetic radiation, but he argued in the end that "the result is independent of the mechanism by which the system loses energy", and that "inertial mass is associated with all forms of energy" in that a change in energy will result in a change in inertial mass by the amount $m = E/c^2$ (CP, TR, p. 269), which would coincide with the previous $E = mc^2$. By 1911, when Einstein was already preparing ideas for a general theory of relativity, he produced another paper ("Über den Einfluß der Schwerkraft auf die Ausbreitung des Lichtes"; "On the Influence of Gravitation on the Propagation of Light") in which he showed the gravitational mass of a body (which would be shown to be equivalent to inertial mass in general relativity) to be subject to increase when absorbing energy E by the same amount of E/c^2. Finally, Einstein many years later (and Langevin and Perrin before him) was able to produce a purely dynamical derivation of $E = mc^2$, from $E^2 = m_0^2c^4 + p^2c^2$ (where p = momentum). (ENS, FF)

Below are provided Einstein's arguments of 1905 and 1916. Both are based on the further assumption of the "general validity" of the principle of relativity, i.e. its validity in relation to, beyond the law of the propagation of light, also the conservation of energy:

> Das Relativitätsprinzip fordert, daß der Satz von der Erhaltung der Energie nicht nur bezüglich eines Koordinatensystems K gelte, sondern bezüglich eines jeden Koordinatensystems K', das relativ zu K sich in gleichförmiger Translationsbewegung befindet (... bezüglich jedes "Galileischen" Koordinatensystem). (1916, p. 31)
>
> The principle of relativity requires that the law of the conservation of energy should hold not only with reference to a co-ordinate system K, but also with

> reference to every co-ordinate system K' which is in a state of uniform motion of translation relative to K (... relative to every "Galilean" coordinate system of co-ordinates).

The first argument of 1905 proceeds by showing that the kinetic energy of a body would decrease after its emission of a light signal which must have thus carried the lost energy away. Since kinetic energy is a reflection of inertial mass, this means that mass itself must have decreased -- part of it being "carried away" by light.

> Ein system von ebenen Lichtwellen besitze, auf das Koordinatensystem (x, y, z) bezogen, die Energie [E]; die Strahlrichtung (Wellennormale) bilde den Winkel ϕ mit der x-Achse des Systems. Führt man ein neues, gegen das System (x, y, z) in gleichförmiger Paralleltranslation begriffenes Koordinatensystem (ξ, η, ζ) ein, dessen Ursprung sich mit der Geschwindigkeit v längs der x-Achse bewegt, so besitzt die genannte Lichtmenge -- im System (ξ, η, ζ) gemessen -- die Energie:
>
> Let a system of plane light waves possess, in relation to the coordinate system (x, y, z), energy E; let the direction of the ray (the wave-normal) form an angle ϕ with the x-axis of the system. If one introduces a new coordinate system (ξ, η, ζ) moving in uniform parallel translation with respect to the system (x, y, z), and whose origin moves along the x axis with velocity v, then this quantity of light -- measured in the system (ξ, η, ζ) -- possesses the energy
>
> $$E^* = E\frac{1-\frac{v\cos\varphi}{c}}{\sqrt{1-\frac{v^2}{c^2}}}$$
>
> ... Von diesem Resultat machen wir im folgenden Gebrauch. Es befinde sich nun im System (x, y, z) ein ruhender Körper, dessen Energie -- auf das System (x, y, z) bezogen -- E_0 sei. Relativ zu dem wie oben mit der Geschwindigkeit v bewegten System (ξ, η, ζ) sei die Energie des Körpers H_0.
>
> ... Of this result let us make the following use. Let there be a stationary body in the system (x, y, z), and let its energy -- in relation to the system (x, y, z) -- be E_0. Let the energy of the body relative to the system (ξ, η, ζ) moving as above with the velocity v, be H_0.
>
> Dieser Körper sende in einer mit der x-Achse den Winkel ϕ bildenden Richtung ebene Lichtwellen von der Energie L/2 (relativ zu (x, y, z) gemessen) und gleichzeitig eine gleich große Lichtmenge nach der entgegengesetzten Richtung. Hierbei bleibt der Körper in Ruhe in bezug auf das System (x, y, z). Für diesen Vorgang muß das Energieprinzip gelten und zwar (nach dem Prinzip der Relativität) in bezug auf beide Koordinatensysteme. Nennen Wir E_1 bez. H_1 die Energie des Körpers nach der Lichtaussendung relativ zum System (x, y, z) bez. (ξ, η, ζ) gemessen, so erhalten wir mit Benutzung der oben angegebenen Relation:
>
> Let this body send out, in a direction forming an angle ϕ with the x axis, plane waves of light of energy L/2 (measured relative to (x, y, z)), and simultaneously an equal quantity of light in the opposite direction.

Meanwhile the body remains at rest with respect to system (x, y, z). The principle of energy must hold for this process, and in fact (according to the principle of relativity) with respect to both coordinate systems. If we name the energy of the body after the emission of light E_1 or H_1 [respectively], measured relatively to the system (x, y, z) or (ξ, η, ζ) [respectively], then we obtain by employing the relation given above:

$$E_0 = E_1 + \left(\frac{L}{2} + \frac{L}{2}\right) \text{ and } H_0 = H_1 + \left(\frac{L}{2} \frac{1 - \frac{v}{c}\cos\varphi}{\sqrt{1 - \frac{v^2}{c^2}}} + \frac{L}{2} \frac{1 + \frac{v}{c}\cos\varphi}{\sqrt{1 - \frac{v^2}{c^2}}} \right) = H_1 + \frac{L}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Durch Substraktion erhält man aus diesen Gleichungen:

By subtraction one obtains from these equations

$$(H_0 - E_0) - (H_1 - E_1) = L\left(\frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} - 1 \right)$$

Where $(H_0 - E_0)$ is the difference between the moving and resting system's measurement of the energy value of the body before the emission of light, and $(H_1 - E_1)$, the difference after the emission of light. Note that the above derived formula is already identical to that earlier one for the kinetic energy of a body (L takes the place of m_0c^2).

Die beiden in diesem Ausdruck auftretenden Differenzen von der Form H - E haben einfache physikalische Bedeutungen. H und E sind Energiewerte desselben Körpers, bezogen auf zwei relativ zueinander bewegte Koordinatensysteme, wobei der Körper in dem einen System (System (x, y, z)) ruht. Es ist also klar, daß die Differenz H - E sich von der kinetischen Energie K des Körpers in bezug auf das andere System (System (ξ, η, ζ)) nur durch eine additive Konstante C unterscheiden kann, welche von der Wahl der willkürlichen additiven Konstanten der Energien H und E abhängt.

The two differences of the form H - E occurring in this expression have simple physical significations. H and E are energy values of the same body in relation to two coordinate systems moving relative to one another, of which the obdy is at rest in one system (system (x, y, z)). Thus it is clear that the difference H - E can be distinguished from the kinetic energy K of the body, in relation to the other system (system (ξ, η, ζ)), only by an additive constant C, which depends on the choice of the arbitrary additive constants of H and E.

In other words, the difference in measurement of the body's energy between the two "points of view" amounts just to its kinetic energy in the one moving relative to it.

Wir können also setzen:

We can thus posit:

$$H_0 - E_0 = K_0 + C$$
$$H_1 - E_1 = K_1 + C$$

da C sich während der Lichtaussendung nicht ändert. Wir erhalten also:

because C during the emission of light does not change. We obtain thus:

$$K_0 - K_1 = L\left(\frac{1}{\sqrt{1-\frac{v^2}{c^2}}} - 1\right)$$

Die kinetische Energie des Körpers in bezug auf (ξ, η, ζ) nimmt infolge der Lichtaussendung ab, und zwar um einen von den Qualitäten des Körpers unabhängigen Betrag. Die Differenz K_0 - K_1 hängt ferner von der Geschwindigkeit ebenso ab wie die kinetische Energie des Elektron.

The kinetic energy of the body in relation to (ξ, η, ζ) diminishes as a result of the emission of light, and indeed by an amount independent of the properties of the body. Moreover, the difference K_0 - K_1, just like the kinetic energy of the electron, depends on the velocity.

Unter Vernachlässigung von Größen vierter und höherer Ordnung können wir setzen:

Neglecting magnitudes of fourth or higher orders we can posit:

$$K_0 - K_1 = \frac{L}{c^2}\frac{v^2}{2}$$

Aus dieser Gleichung folgt unmittelbar: Gibt ein Körper die Energie L in Form von Strahlung ab, so verkleinert sich seine Masse um $L/[c]^2$.

It follows from this equation that, if a body gives off energy L in the form of radiation, its mass diminishes by $L/[c]^2$.

The above formula is thus the one for kinetic energy in classical mechanics: since L/c^2 is the change in mass Δm, $(L/c^2)(v^2/2) = \Delta m(1/2)v^2 = 1/2\Delta mv^2$, which is the kinetic energy lost. Since kinetic energy depends on mass, its change can only mean a corresponding change in mass, as shown in Δm above. Thus, "Die Masse eines Körpers ist ein Maß für dessen Energieinhalt" ("The mass of a body is a measure of its energy-content"). Or, mass and energy are mutually convertible. This conclusion emerges entirely from the equations themselves, as seen above, in the deductive manner.

$E = mc^2$ coincides here because $\Delta m = \Delta E/c^2$, so $\Delta E = \Delta mc^2$. Einstein did not make this step at this point, however. Note that, with the more conservative $\Delta m = \Delta E/c^2$, it is apparent that "it is the finiteness of the speed of light c that is responsible for the equivalence between mass and energy. Should c be infinite, as assumed in [Newtonian-Galilean approximation], any change in energy would result in a zero change of mass." (ENS) Einstein writes of this most important consequence of special relativity (1916): "The pre-relativity physics recognised two conservation laws of fundamental importance, namely, the law of the conservation of energy and the law of the conservation of mass; these two fundamental laws appeared to be quite independent of each other" ("Die vorrelativistische Physik kennt zwei Erhaltungssätze von grundlegender Bedeutung, nämlich den Satz von der Erhaltung der Energie und den Satz von der Erhaltung der Masse; diese beiden Fundamentalsätze erscheinen als ganz unabhängig voneinander"; p. 31). This is then due to the fact that the speed of light in pre-relativity physics is taken as it appears to our eyes, i.e. *as it seems*: infinite. The principle of conservation of mass is thus a *seeming*. "Through the theory of relativity they have been united into one law" ("Durch die Relativitätstheorie werden sie zu einem Satze verschmolzen"; ibid.). This happens then when the speed of light is grasped precisely, so that the conservational principle also comes *into precision.* Elsewhere Einstein notes that the principle of conservation of mass admitted in Newtonian mechanics is only valid for a system whose energy remains constant (1910, p.; 1916, p. 32).

The derivation Einstein gives in (1916; p. 31 - 2) is of the same kind but in reverse; it deals with the *increase* of kinetic energy after absorption of radiation -- that is, while the previous argument demonstrates the conversion of mass to energy, here the demonstration is for the conversion of energy to mass -- and it not only explicitly uses the new relativistic *eidos* for kinetic energy, but also derives $E = mc^2$ directly.

Ein mit der Geschwindigkeit v fliegender Körper, der in Form vom Strahlung die Energie E_0 aufnimmt [von einem mit dem Körper bewegten Koordinatensystem aus beurteilt], ohne hierbei seine Geschwindigkeit zu ändern, erfährt dabei eine Zunahme seiner Energie um den Betrag [von einem relativ zu dem bewegten Koordinatensystem ruhende Koordinatensystem aus beurteilt]:	A body moving with the velocity v, which absorbs an amount of energy E_0 in the form of radiation [as judged from a co-ordinate system moving with the body] without suffering an alteration in velocity in the process, has, as a consequence, experienced an increase of its energy by the amount [as judged from a co-ordinate system at rest relative to the moving coordinate]

$$\frac{E_0}{\sqrt{I - \frac{v^2}{c^2}}}$$

Die gesuchte Energie des Körpers ist also dann mit Rücksicht auf den vorher angegebenen Ausdruck für die kinetische Energie gegeben durch:	In consideration of the expression given above for the kinetic energy of the body, the sought-after energy of the body is given by

$$\frac{\left(m + \frac{E_0}{c^2}\right)c^2}{\sqrt{I - \frac{v^2}{c^2}}}$$

Since $[mc^2/\sqrt{(1 - (v^2/c^2))}] + [E_0/\sqrt{(1 - (v^2/c^2))}] = [(mc^2 + E_0)/\sqrt{(1 - (v^2/c^2))}]$; factoring out c^2 from $(mc^2 + E_0)$, we get $(m + (E_0/c^2))c^2$.

Der Körpers hat also dann dieselbe Energie wie ein mit der Geschwindigkeit v bewegter Körper von der Masse $(m + (E_0/c^2))$. Man kann also sagen: Nimmt ein Körper die Energie E_0	Thus the body has the same energy as a body of mass $(m + (E_0/c^2))$ moving with a velocity v. Hence we can say: If a body takes up an amount of energy E_0, then

auf, so wächst seine träge Masse um E_0/c^2.	its inertial mass increases by an amount E_0/c^2.

This is so of course from that "perspective" at rest with respective to the "perspective" moving with the body. Now, if we return to its form as the *eidos* for kinetic energy:

$$\frac{mc^2 + E_0}{\sqrt{I - \frac{v^2}{c^2}}}$$

so sieht man, daß die Form mc^2, die uns schon vorhin auffiel, nichts anderes ist als die Energie, welche der Körper schon besaß [von einem mitbewegeten Koordinatensystem aus beurteilt], bevor er die Energie E_0 aufgenommen hatte.	we see that the term mc^2, which has hitherto attracted our attention, is nothing else than the energy possessed by the body [as judged from a co-ordinate system moving with the body] before it absorbed the energy E_0.

Since, in both cases of change of mass experienced by the body, this body is considered from the "perspective" not moving with it, what is being described is that a *moving* body, upon absorbing or emitting a light ray of energy E, would experience an increase or decrease of mass by E/c^2. That is, in both cases, a moving body's loss or increase of mass is only measured by means of, or deduced from, the increase or loss of its kinetic energy. Since, as said, so much of the parallel between mysticism and physics depends on this mass-energy interchangeability as implied by special relativity, we must be clear as to what this interchangeability means in two respects. Firstly, these two considerations do not yet suffice in confirming a complete derivation of matter from energy as required in mysticism. As Francisco Flores explains:

> A common misconception surrounding $E = mc^2$ [in special relativity] is that it entails that the entire rest-mass of a body can become energy. Strictly speaking, mass-energy equivalence only entails that a change in the rest-energy of a body is invariably accompanied by a corresponding change in the rest-mass of the body. For example, a body may lose a bit of its mass because it radiates a bit of energy. The stronger claim that a body may lose all of its rest-mass as it radiates energy is not a consequence of [special relativity]. However, this stronger claim is very well confirmed by experiments in atomic physics. Many particle-antiparticle collisions have been observed, such as collisions between electrons and positrons, where the entire mass of the particles is radiated away as energy in the form of light. Nevertheless, [special relativity] leaves open the possibility that a form of matter exists whose mass cannot become energy. This is significant because it emphasizes that mass-energy equivalence is not a consequence of a theory of matter; it is instead a direct consequence of changes to the structure of spacetime imposed by [special relativity].

The second point concerns the meaning of the energy-matter equivalence.

> There are two main philosophical interpretations of mass-energy equivalence. According to one common interpretation, $E = mc^2$ implies that mass and energy, which are treated as distinct properties in Newtonian physics, are actually the same. I will refer to this view, which is the weaker of the two, as the same-property interpretation hereafter. The second interpretation of mass-energy equivalence is that it entails that there is only

> one fundamental stuff in the world. I will call this view the one-stuff interpretation hereafter.

The two examples of the weaker "same property" interpretation that Flores provides are from Eddington (*Space, Time, and Gravitation*, 1929) and Torretti (*Relativity and Geometry*, 1983).

> Eddington states that "it seems very probable that mass and energy are two ways of measuring what is essentially the same thing, in the same sense that the parallax and distance of a star are two ways of expressing the same property of location" (ibid., p. 146). According to Eddington, the distinction between mass and energy is artificial. We treat mass and energy as different properties of physical systems because we routinely measure them using different units. However, one can measure mass and energy using the same units by choosing units in which c = 1, i.e., units in which distances are measured in units of time (e.g., light-years). Once we do this, Eddington claims, the distinction between mass and energy disappears.

c is thus a conversion factor both between space and time and between mass and energy.

> For Torretti, the apparent difference between mass and energy is thus an illusion that arises from "the convenient but deceitful act of the mind by which we abstract time and space from nature" (ibid., p. 307, fn. 13).... [H]e seems to be suggesting that in our perception of the world spatial and temporal dimensions merely *appear* to be distinct. We perceive spatial intervals as different in kind from temporal intervals. Consequently, we use different types of units to measure spatial and temporal intervals, which has the consequence that mass and energy have different types of units.

Time and space only *appear* to be distinct since time can be treated, by means of the imaginary number, as simply an extra spatial dimension in the Euclidean sense (which is not completely true, as noted). These weaker interpretations are inadequate for mysticism because they "draw no further ontological conclusions from mass-energy equivalence." For that we need the stronger interpretation. The stronger kind however consists of two types; only the first is what Einstein has really thought, but it is not this that will underlie, as its scientific equivalent, traditional mysticism.

> Interpretations in the second group establish a connection between the terms "mass" and "energy," which are again treated as terms designating properties, and the two basic constituents in the ontology of physics: matter and fields. The equivalence of mass and energy is then taken to show that we can no longer distinguish between matter and fields. Einstein and Infeld [*The Evolution of Physics*, 1938] offer a clear articulation of this interpretation. According to Einstein and Infeld, in pre-relativistic physics one can distinguish matter from fields by their properties. Specifically, matter has energy and mass, whereas fields only have energy. Since mass and energy are distinct in pre-relativistic physics, there are physical criteria that allow us to distinguish matter from fields qualitatively. So it is reasonable to adopt an ontology that contains both matter and fields. However, in relativistic physics, the qualitative distinction between matter and fields is lost because of the equivalence of mass and energy. Consequently, Einstein and Infeld argue, the distinction between matter and fields is no longer a qualitative one in relativistic physics. Instead, it is merely a quantitative difference, since "matter is where the concentration of energy is great, field where the concentration of energy is small" (ibid., p. 242). Thus... mass-energy

> equivalence entails that we should adopt an ontology consisting only of fields.

It is the second type with which traditional, philosophic mysticism converges.

> According to Russell, "a unit of matter tends more and more to be something like an electromagnetic field filling all space, though having its intensity in a small region" ["The Ultimate Constituents of Matter", 1915, in *Mysticism and Logic*, p. 121).... For Russell, these considerations suggest that "mass is only a form of energy, and there is no reason why matter should not be dissolved into other forms of energy. It is energy, not matter, that is fundamental in physics" [*Human Knowledge: Its Scope and Limits*, 1948, p. 291]. Russell is proposing that mass is reducible to energy in the sense that the world consists only of energy.... Several physicists have held a similar position, though this view is less common now. For example, after a discussion on particle-antiparticle annihilation experiments in 1951, Wolfgang Pauli states: "Taking the existence of all these transmutations into account, what remains of the old idea of matter and of substance? The answer is energy. This is the true substance, that which is conserved; only the form in which it appears is changing" (1951, p. 31).

This is indeed the scientific expression of how "mystics" in history (at the immanentist level, at least) have conceived of the substratum of all being, of consubstantiality, of "Being is One": Thales' water, Anaximenes' air, Anaximander's apeiron, Daoists' Dao, Neoconfucians' and Sung dynasty Yijing metaphysicians' *qi* (air), are all ancient (functional) equivalents of the underlying, unchanging, and eternal "energy". It seems that most New Agers fall in here, too.

> Russell and Pauli's interpretations are, despite the superficial similarity, importantly different from Einstein and Infeld's. Russell (in some places) and Pauli both treat the term "energy" as though it designates a substance, whereas Einstein and Infeld clearly regard energy as a property. This is an important difference. Treating energy as a term designating a substance is now widely regarded as a remnant of an untenable nineteenth century view....

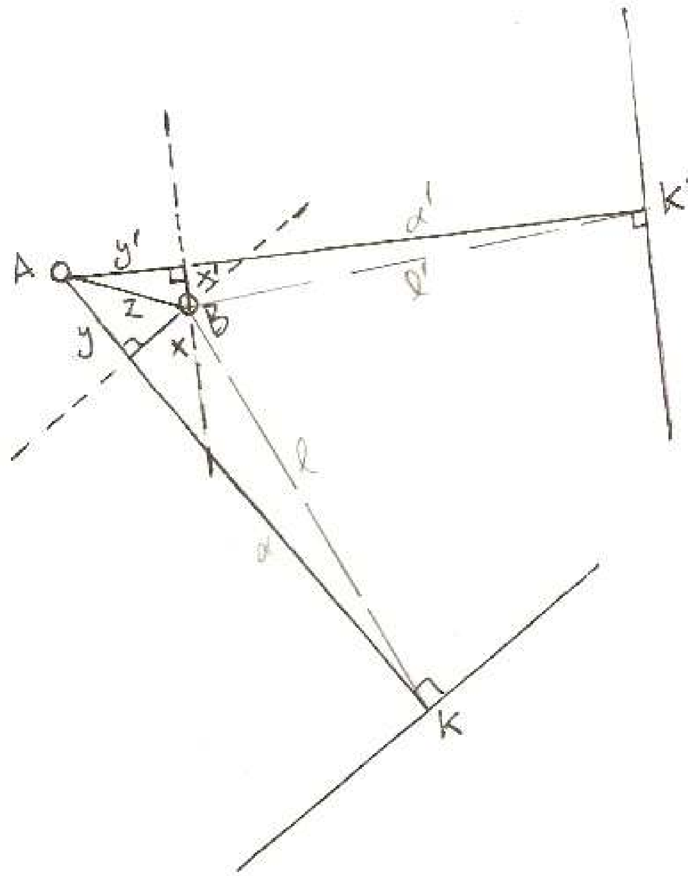
The traditional mystics' conception (when updated to its equivalent scientific articulation) is based on the "substance interpretation" of energy and a non-distinction between mass and matter, and between force and energy. Mass is the quantity of matter, measured in inertia, and "to all force corresponds necessarily an energy of interaction between material objects." (TD p. 74) *Scientific mysticism* therefore has to be based on Einstein's "field interpretation" of energy, where matter becomes our perception of the property of inertia/ mass which a high concentration of energy in a local region of the field acquires. As Max Born summarizes: "Matter in the widest meaning of the word (including light and other forms of pure energy, in the language of classical physics) has two fundamental qualities: inertia, measured by its mass, and the capability of performing work, measured by its energy. These two are strictly proportional to one another. Wherever electric and magnetic fields or other effects lead to intense accumulations of energy, they are accompanied by inertia. Electrons and atoms are examples of enormous concentrations of energy." (Einstein’s *Theory of Relativity*, Dover Publications, 1962, p. 286; quoted in ENS). When combined with the field-conception of energy and clarified by the distinction between mass and matter and between force and energy, the "stronger claim" of $E = mc^2$ thus provides the scientific version of immanent mysticism (immature philosophy) -- the scientific version of the apeiron or *qi*:

> Modern cosmology suggests... that all the "matter" which surrounds us, and of which we are made, did not exist during the first stages of the expansion of the universe, and was created from the energy that was stored in the continuous field (somewhat similar to the electromagnetic field) present in all of space. (TD, ibid.)
>
> La cosmologie moderne suggère... que tout la "matière" qui nous entoure, et dont nous sommes faits, n'existait pas pendant les premiers stades de l'expansion de l'univers, et a été créée à partir de l'énergie qui était stockée dans un champs continu (un peu semblable au champ électromagnétique) présent dans tout l'espace.

Relativity in the overall course of the evolution of consciousness. Special relativity represents the attempt of consciousness to enlarge the range of its representation of reality: specifically, as said in the beginning, from the single position-bound perspective to a universal perspective, an overarching perspective, the perspective of the Heaven (in Zhuangzi's saying). Einstein's discovery of the relativity of simultaneity was the moment when things were at last looked at "from above", rather than from the same surface as they were located:

> Let us assume that we were asked whether, apart from frames of all kinds, the two events really occurred simultaneously. Unfortunately, this question has no more sense than the one whether, regardless of all points of observation, the two stars are really aligned. The fact is that simultaneity depends not only on the two events but also on the frame from which we observe them, just as alignment of the two stars depends not only on their position, but also on the point from which they are observed. (LR, p. 37)

In other words, "simultaneity" is a category of thought produced by, proper to, a specific mode of perspective, that of a single inertial frame. Simultaneity's being a valid category of reality means the conservation of the units of time. Once the perspective changes, such as through enlargement of its horizon to include the points of view of all inertial frames in uniform translation movement relative to one another, then the very concept of "simultaneity" disintegrates -- as it is a non-existent situation from the higher perspective -- to be replaced (together with the conservation of the units of space) by something else: the conservation of space-time distance; this is just as the "alignment of stars" becomes a meaningless concept when astronomy attempts to account for all positions of observation. It is, again, not the units of space that are conserved, nor the units of time, but the units of space-time, just as it is not mass that is conserved, nor energy, as is thought in pre-relativistic mechanics, but mass-energy. This is exactly like that parable of blind men feeling the elephant: when people disagree about the timing and position of an event, it is because they are seeing it from different inertial frames, just as the blind men disagree about the shape of the elephant (the one feeling the leg thinks it is like a tree, the one feeling the tail thinks it is like a rope) because they are feeling different parts of the elephant. When the blind men get cured and see the whole elephant -- transcend their limited positions and take in the whole elephant at once -- they realize that before they know only parts of the elephant and so disagree. (Relativity certainly didn't teach that "truth" is relative and not absolute, but rather that the absolute truth is of a higher level and the truth hitherto known is only partial truth.) In a more comprehensive perspective, the previous identities of classical mechanics, space, time, force (energy), and matter (mass) cease their existence as such, and turn out to be aspects of spacetime and mass-energy. Consider the figure below.



x and x' = time differences dt and dt' between events A and B for perspectives (inertial coordinate systems) K and K'; y and y' = space distances. As long as K never thinks of K' (or assumes that K' will see exactly what it itself sees) when considering the distance between A and B, it will assume that the distance between A and B are an invariant x in terms of time and an invariant y in terms of space, rather than their invariant "combination" together, z, which is the real invariant for K and K' (what both will see). The illusion of the invariant, independent existence of x and of y is due to the obscuring effect of the necessary reduction in representation by one dimension (the alignment of stars is due to the projection of their three dimensional distance onto the two dimensional plane in the representational process of our vision). The actual distance in the higher dimension (z) is thus overlooked -- until one induces it by looking at it seriously from another position.

Consciousness always evolves from a subjective, position-limited perspective to an objective, more comprehensive, position-unlimited "universal" perspective, in whatever domains. In the domain of ethics (or values), we see the same evolution: consciousness begins, in its perception and judgment of the "good" or the "right", with a purely subjective state occupied entirely by a single perspective (egocentrism: whatever is good for me is good; or sociocentrism: whatever is good for my group is good: the conventional and preconventional stages in Kohlberg's moral development schema), and then evolves gradually toward an objective state comprehending eventually all possible perspectives (the worldcentric view, in which the real good has to be good for all: first the "Golden Rule" ["Do onto others as you would others do onto you"] with its flaw [i.e. people have different tastes] and finally the "ideal position" such as in John Rawls' *A Theory of Justice*: the post-conventional stage of moral development). So in the realm of physics (or facts) we see in consciousness' progress from pre-relativistic classical mechanics to relativity a movement from the representation of physical or geospheric phenomena which is subjective, bound to a single perspective in which the units of space and time are thus fixed, i.e. have a precise, absolute meaning, to that representation which comprehends all perspectives, is valid for all of them, but in which the conserved units of space and time cease as valid categories of thought to give rise to a new category of thought, the conserved units of space-time.

The higher disillusionment with the flux of time. Minkowski's 1908 presentation begins with the statement: "Von Stund an sollen Raum für sich und Zeit für sich völlig zu Schatten herabsinken und nur noch eine Art Union der beiden soll Selbständigkeit bewahren." ("From now on space by itself and time by itself must fully fade into the shadows and only a sort of union of the two will preserve an independent reality.") Now think about what this really means, think about the full implication of the Minkowski "world", that "four-dimensional bloc from which all temporal flux is banished." (TD, p. 57; "... 'la réalité relativiste' doit être pensée dans l'espace-temps, c'est-à-dire dans un bloc quadridimensionnel d'où tout flux temporel est banni...") Damour uses the example of insects crawling on a flat surface to illustrate "the necessity to think of the 'space-time bloc' outside all temporal flux" (ibid.). An easier visualization of the Minkowski "world" is

> the habitual idea that the "world" of insects living on the floor is made of a succession of "instants", each representing the "state of the floor" at each instant of time. Each "instant" describes the configuration on the floor, during the instant under consideration, of all the insects that live there. This spatial configuration, at a particular instant, can be completely described by a photograph, a "instantaneous negative", of the surface of the floor. Then, the three-dimensional space-time of the insects living on the floor is obtained by piling up, vertically, these successive negatives, of which each represents the state of space at a moment of time.... The height of each negative in this pile is proportional to the date corresponding to the negative.
>
> To each insect corresponds a "spot" on each photograph of the pile, and to each instant of time corresponds a spot [made] by the insect. [This "spot" is how relativity has transformed the *Geschehen* into a *Sein*.] The life of each insect thus defines a continual succession of spots, which traces out a tube (a thick line) in space-time... If the insect stays at rest on the floor, its "spacetime tube"... ascends vertically, i.e. in an orthogonal fashion with respect to the horizontal directions which represent the "space" on which the insects live. On the other hand, if the insect moves, its spacetime tube will incline from the vertical direction. The faster it moves, the greater the inclination of the tube. (TD, p. 54)
>
> l'idée habituelle que le "monde" d'insectes vivant sur le plancher est fait d'une succession d'"instantané", représentant chacun l'"état du plancher" à chaque instant du temps. Chaque "instantané" décrit la disposition sur le plancher, à l'instant considéré, de tous les insectes qui y vivent. Cette disposition spatiale, à un instant déterminé, peut être complètement décrite par une photographie, un 'cliché instantané', de la surface du plancher. Alors, l'espace-temps tridimensionnel des insectes vivant sur le plancher est obtenu en *empilant*, verticalement, la suite continue de ces clichés, dont chacun représente l'état de l'espace à un moment du temps.... La hauteur de chaque cliché dans cet empilement est proportionnelle à la date correspondant à ce cliché.
>
> À chaque insecte correspond une "tache" sur chaque photographie de l'empilement, et à chaque instant du temps correspond une tache par insecte.

> La vie de chaque insecte définit ainsi une succession continue de taches, qui trace un tube (une ligne épaisse) dans l'espace-temps.... Si l'insecte reste au repos sur le plancher, son "tube d'espace-temps"... s'élève verticalement, c'est-à-dire de façon orthogonale aux directions horizontales qui représentent l'"espace" où vivent les insectes.En revanche, si l'insecte se déplace, son tube d'espace-temps sera incliné par rapport à la verticale. Plus il se déplace vite, plus l'inclinaison du tube augmente.

The insect reaches the "limiting speed" therefore when its spacetime tube coincides completely with the horizontal direction of the floor: the speed of light. Normally, of course, an insect moves slowly and back and forth and round and about, so that its spacetime tube -- the trajectory that it traces out in spacetime, its four-dimensional spacetime trajectory -- is spiral-like and so on. In reality, "this tube takes up considerably much more place in time than in space. In effect, when time is measured in seconds and space in light-seconds, this tube has a (temporal) height of several billions of seconds, while its (spatial) width is only a few billionth of a light-second" (p. 63; "... ce tube occupe une place beaucoup plus considérable dans le temps que dans l'espace. En effet, en mesurant.... les durées en secondes, et les distances en secondes-lumière, ce tube a une hauteur (temporelle) de quelques *milliards* de secondes, alors que sa largeur (spatiale) est seulement de quelques *milliardièmes* de secondes-lumière"). The spacetime trajectory is almost an one-dimensional line within a three dimensional spacetime. The essential point here is that this line is purely spatial: "nothing in the formalism of special relativity corresponds to the idea of 'now', i.e. to the existence of a privileged 'instant' describing the 'present'" (p. 57). We think of "time" as "in flux," i.e. "the past no longer exists, and the future not yet exists, and has therefore no actual reality" (p. 56). But if there is no negative privileged in the pile, no "spot" privileged in the tube, there is no real "present", but, in fact, the entire spacetime trajectory exists all at once, past, present, and future exist all at once, just as we say all things in space "at present" exist all at once. The consistent application of the principle of relativity therefore leads to the enlightenment or disillusionment that "time" as we know it, i.e. a flux from the existent to the not-yet existent, with the existent passing into non-existence and the not-yet existent coming into existence, *together with* its irreversibility, is just so much illusion, the effect of our limited dimensionality.

> Space-time describes the entirety of the history of reality *sub specie aeternitatis*, just as a musical score describes the entirety of a musical piece. The score "exists" in a "static" fashion, although it describes something which is generally comprehended by the human mind in the form of a temporal flux. The reader might think perhaps that this comparison suggests rather that a "static" spacetime is not any more capable of taking account of the movement of reality than the vision "en bloc" of a musical piece could correctly comprehend the essence of what music is.

> L'espace-temps décrit l'ensemble de l'histoire de la réalité *sub specie aeternitatis*, comme une partition décrit l'ensemble d'une oeuvre musicale. La partition "existe" de façon "statique", bien qu'elle décrive quelque chose qui est généralement appréhendé par l'esprit humain sous la forme d'un flux temporel. Le lecteur pensera peut-être que cette comparaison suggère plutôt qu'un espace-temps "statique" n'est plus capable de rendre compte de la mouvance du réel que la vision "en bloc" d'une oeuvre musicale ne saurait correctement appréhender l'essence de ce qu'est la musique.

Damour then cites Mozart's experience with musical composition.

> "The work is then finished in my head, or almost so, even if it is a long piece, and I can embrace its entirety all at once like a painting or a statue. In

my imagination, I don't hear the work in its flow such as it must run in succession, but I take in its entirety in a single bloc, so to speak..."

"L'oeuvre est alors achevée dans mon crâne, ou vraiment tout comme, même si c'est un long morceau, et je peux embrasser le tout d'un seul coup d'oeil comme un tableau ou une statue. Dans mon imagination, je n'entends pas l'oeuvre dans son écoulement, comme ça doit se succéder, mais je tiens le tout d'un bloc, pour ainsi dire...."

As the quotation shows, a great musician can transcend the habitual way in which the simple mortals understand music, in order to "super-hear" "in a single bloc", outside any temporal flow. The structure of the theory of relativity suggests that if one could liberate himself of the thermodynamic and biological constraints that condition us, in our everyday life, to live reality in the form of "temporal flux", one could, by analogy, "super-live" our life "in a single bloc", as being part of the four-dimensional spacetime bloc of Minkowski (p. 57 - 8).

Comme le montre cette citation, un grand musicien peut transcender la façon habituelle dont les simples mortels appréhendent la musique pour la "super-entendre" "d'un bloc", hors de tout écoulement temporel. La structure de la théorie de la relativité suggère que si l'on pouvait s'affranchir des contraintes thermodynamiques et biologiques qui nous conditionnent, dans la vie de tous les jours, à vivre la réalité sous la forme d'un "flux temporel", on pourrait, par analogie "super-vivre" notre vie "d'un bloc", comme faisant partie du bloc espace-temps quadridimensionnel de Minkowski.

The twin paradox and the transition to general relativity. Paul Langevin expressed in 1911 the earliest version of the "twin paradox," which has now become a famous associate of special relativity. The established version goes something like this (JLS, DP). There are two twins, A and B, who were born on Earth. Twin A gets in a spaceship which quickly accelerates to a large velocity (say 0.9 the speed of light) relative to the Earth. This twin travels for a period of proper time, say 10 years, quickly turns around, and returns to Earth where he decelerates and stops. Now, disregarding the minor variations caused by the acceleration and deceleration, the traveling twin will have aged only 20 years, while the twin who has stayed on Earth, B, will have aged roughly 46 years, according to the prediction of relativity:

```
event a              event b

twin A @ t = 0       twin A @ t1 = 10 yrs (distance btw a & b = ds)

event a'             event b'

twin B @ t'= 0       twin B @ t1'= x yrs (distance btw a'& b'= ds')
```

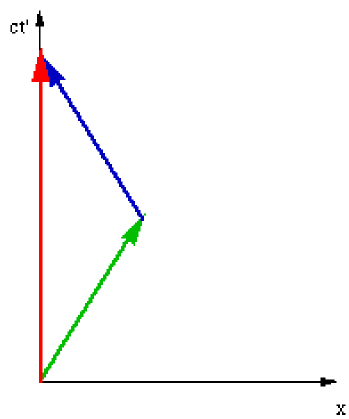
The task of "prediction" is to find dt', i.e. (t1' - t') (or t1' since t' = 0), given that ds = ds', and since twin A considers herself (is by her own clock) "at rest".

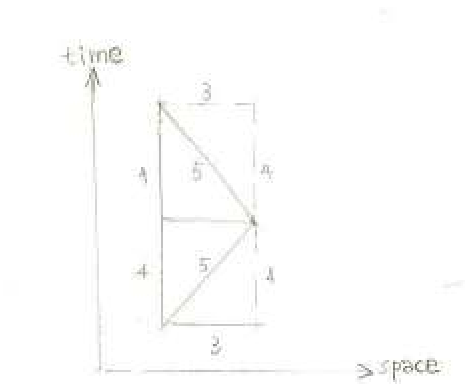
$ds^2 = (b - a)^2 = dx^2 + dy^2 + dz^2 - c^2dt^2 = c^2(10\ yrs)^2$
$ds'^2 = (b' - a')^2 = dx'^2 + dy'^2 + dz'^2 - c^2dt'^2$
$= ([0.9cdt']^2 + 0 + 0) - c^2dt'^2$
$= 0.9c^2dt'^2 - c^2dt'^2$
$= ([0.9c]^2 - [1c]^2)dt'^2$
$= (0.81c^2 - 1c^2)dt'^2$
$= (0.19c^2)(dt'^2)$

Now, $(c^2(10\text{ yrs})^2)/c^2 = ((0.19c^2)(dt'^2))/c^2$
$100 = 0.19(dt'^2)$
$dt'^2 = 100/0.19 = 526...$
$\sqrt{526} = 22.9$
23 x 2 = 46 years corresponding to A's round trip voyage.

So far "this is odd but not paradoxical. The paradox is in the objection that if the effects of absolute motion are unobservable and only relative motion can be detected, one might just as well say that the earth with B on it went away from the spaceship and came back, so that A would be younger. Thus the argument seems to require A on her return to be both older and younger than B." (DP, p. 297)

"The key that unlocks the twin paradox is the fact that A is obliged to move nonuniformly during at least part of her trip, while B does not accelerate at all. A detailed analysis taking this acceleration into account... shows that A does indeed return younger, just as predicted by a naive application of special relativity" (ibid.). In other words, the acceleration and deceleration that A needs to leave the earth, turn around, and stop by the earth again, have taken the phenomenon beyond the framework of uniform translation movement between inertial frames within which, only, special relativity is valid. The solution of the twin paradox is thus general relativity.





The triangle representing spacetime path of the two twins as drawn by the twin remaining on Earth. The elapsed time to the traveling twin (his proper time) is less since the length of the bent path is *less* than the length of the straight line. "This 'inequality of the triangle of spacetime' is the reverse of the inequality of the triangle of ordinary Euclidean space, for which the sum of the sides is longer than the [straight line]. This difference is due to the particular form of the chronogeometry of spacetime, where the Pythagorean theorem contains a negative sign for the squares of the sides of a right triangle which are directed in time." (TD, p. 65: "Cette 'inégalité d'un triangle d'espace-temps' est en sens contraire de l'inégalité d'un triangle d'un espace euclidien ordinaire, pour lequel la somme des côtés est plus longue que le troisème côté. Cette différence est due à la forme particulière de la chronogéométrie de l'espace-temps, où le théorème de Pythagore contient un signe moins pour les carrés des côtés d'un triangle rectangle qui sont dirigés dans le temps.") For example, while the twin at rest has travelled 0 - 8 = -8 interval of space time, the travelling twin has travelled (3 - 5) + (3 - 5) = -4 (spacetime distance = spatial distance - time difference).

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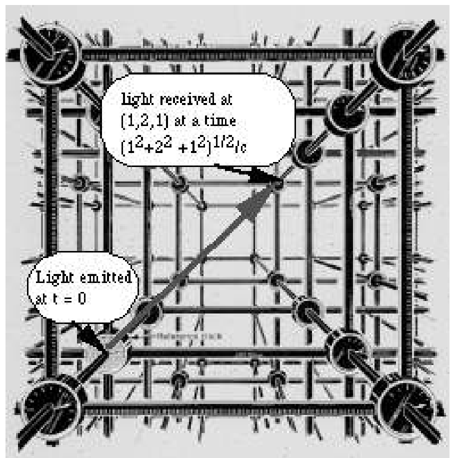
- 2005 is physics year. (The European Nuclear Society) (ENS)
- Einstein Light: original references (EL)

Footnotes:

1. "Galileo's characters in the *Dialogue* do not mention juggling, nor do they say explicitly that there is no way that a sailor in a smoothly moving ship can tell whether the ship is moving or not. What they do say takes up much of the second day of the *Dialogue*. For instance, Salviati says: 'Motion exists as motion and acts as motion in relation to things that lack it, but in regard to things that share it equally, it has no effect and behaves as if it did not exist. Thus, for example, the goods loaded on a ship move insofar as they leave

Venice, go by Corfu, Crete, and Cyprus, and arrive in Aleppo, and insofar as these paces (Venice, Corfu, Crete, etc.) stay still and do not move with the ship; but for the bales, boxes, and packages loaded and stowed on the ship, the motion from Venice to Syria is as nothing and in no way alters their relationship among themselves or to the ship itself; this is so because this motion is common to all and shared equally by all; on the other hand, if in this cargo a bale is displaced from a box by a mere inch, this alone is for it a greater motion (in relation to the box) than the journey of two thousand miles made by them together.' And so on, at very considerable length." (EL)

2. Safko provides a typical approach in today's courses on special relativity to the operational definition of simultaneity (JLS): After placing all clocks at their grid points, we designate one clock as a master clock. Each of the other clocks is stopped and set to read noon plus (distance of clock from the master clock)/(speed of light). When the master clock reads 12 noon, we set off a flashbulb at the master clock's grid location. Each of the clocks on the grid is started when the flash from the bulb reaches that clock. This synchronizes the clocks as shown below:



If the master clock emits a burst of light at t = 0, the other clocks should start at (distance from master clock)/(speed of light). In the Figure, the clock at coordinates (1, 2, 1) should be set to (distance between 0, 0, 0 and 1, 2, 1) divided by c. That is, it should be set to read 2.4/c and started when the flash is seen.

3. As Lorentz wrote in 1915: "The principal cause of my failure [in discovering relativity] was the fact that I was attached to the idea that only the variable t can be considered as the real time and that my local time t' can only be considered as an auxiliary mathematical quantity" (cited in TD, p. 32). He still identified the time of the rest frame with the Newtonian absolute time. This is the same with Poincaré, according to Damour asserts.

Any charge that Einstein plagiarized Lorentz or Poincaré is unfounded. For example, E. T. Whittaker, *A History of Aether and Electricity* (1953). Within the French circle, especially, "the recent years have seen a whole literature flourishing aiming at 'rehabilitating' the contributions of the French mathematician Henri Poincaré to the theory of (special) relativity." (TD, p. 67) For a short specimen of such attempt, see CM. Damour lists others (TD, p. 207): Jean-Paul Auffray, *Einstein et Poincaré* (1999), Jules Leveugle, *La Relativité, Poincaré et Einstein, Planck, Hilbert* (2004), Jean Hladik, *Comment le jeune et ambitieux Einstein s'est approprié la relativité restreinte de Poincaré* (2004).

Marchal, contra Damour, asserts that Poincaré did grasp the relativity of time and space. Poincaré's statements found in *La science et l'hypothèse*, 1902, are: "Non seulement nous n'avons pas l'intuition directe de l'égalité de deux durées, mais nous n'avons même pas celle de la simultanéité de deux événements qui se produisent sur des théâtres différents; c'est ce que j'ai expliqué dans un article intitulé *la Mesure du temps*" (cited by Editors, CP TR, p. 308). In this respect the defenders of Poincaré furthermore attribute the principle of relativity to him ("Le mouvement d'un système quelconque doit obéir aux même lois, qu'on le rapporte à des axes fixes, ou à des axes mobiles entraînés dans un mouvement rectiligne et uniforme"; 1902, p. 135; cited CP TR, p. 308), and point out that he was the first to have introduced the mathematical structure of spacetime in July 1905. (TD, p. 68; more below.) Damour however notes that, for Poincaré, just as for all the others, the principle of relativity was to be deduced from the hypothesis about the structure of matter and the forces acting on it, rather than posited as the point of departure, which was Einstein's approach. Einstein was revolutionary while others were not because he inverted the conventional procedure. "En effet, il part du résultat ('principe de relativité') que les autres essayent de déduire d'hypothèses sur la matière, et il le pose comme un *postulat*, c'est-à-dire comme un point de départ et un outil pour en déduire des résultats généraux concernant la structure de la matière" (TD, p. 69).

Finally, Damour notes, Poincaré never considered his mathematical structure of spacetime to be seriously important for physics (p. 70), being circumscribed by a strict "conventionalism" (p. 71).

4. Because Lorentz and Fitzgerald saw the rest frame of the ether as absolute, objective, "really at rest", they interpreted the mirror system of Michelson and Morley as having "really contracted." Einstein tries explicitly to distinguish himself from Lorentz and Fitzgerald on this matter in his comment on this in 1916, writing: "Die Kontraktion bewegter Körper folgt hier ohne besondere Hypothesen aus den beiden Grundprinzipien der Theorie; zwar ergibt sich als maßgebend für diese Kontraktion nicht die Bewegung an sich, welcher wir keinen Sinn beizulegen vermögen, sondern die Bewegung gegen den jeweilen gewählten Bezugskörper. So ist also für ein mit der Erde bewegtes Bezugssystem der Spiegelkörper von Michelson und Morley nicht verkürzt, wohl aber für ein relativ zur Sonne ruhendes Bezugssystem." (1916, p. 36 - 7) "Here [in relativity] the contraction of moving bodies follows from the two fundamental principles of the theory [the principle of relativity and the constancy of the speed of light], without the introduction of particular hypotheses [i.e. the electronic structures that allow for the contraction of macroscopic body]; and as the prime factor involved in this contraction we find, not the motion in itself, to which we cannot attach any meaning, but the motion with respect to the body of reference chosen in the particular case in point. Thus for a co-ordinate system moving with the earth the mirror system of Michelson and Morley is not shortened, but it is shortened for a co-ordinate system which is at rest relatively to the sun."

5. Consider $x' = (x - vt)/ \sqrt{(1 - v^2/c^2)}$. If c is infinite, then v^2/c^2 becomes infinitely small, $(1 - (v^2/c^2))$ is effectively 1, and its square root, 1. In which case, $(x - vt)/1 = (x - vt)$, which is the Galilean formula.

6. In contemporary notations we would write in the equation for mass-energy equivalence (see below): $E = mc^2 = \gamma m_0 c^2$ (where m is the relativistic mass of the body subject to change when in motion, m_0 its rest mass, and γ the Lorentz factor; note that the total energy E is not subject to change through motion, because it is conserved), and derive the classical equation for kinetic energy from its relativistic form: $E = m_0c^2/ \sqrt{(1 - \beta^2)}$ (where $\beta = (v/c)$). Now β is negligibly small at low velocity, such that $\sqrt{(1 - \beta^2)}$ is approximately $1 - (\beta^2/2)$. $m_0c^2/\sqrt{(1 - \beta^2)}$ then becomes $m_0c^2/(1 - (\beta^2/2))$. Now $1/(1 - (\beta^2/2))$ is approximately $1 + (\beta^2/2)$. Then we have $m_0c^2(1 + (\beta^2/2)) = m_0c^2 + 1/2\, m_0c^2\beta^2 = m_0c^2 +1/2\, m_0c^2(v^2/c^2) = m_0c^2 + 1/2\, m_0v^2$, i.e. the energy of the rest mass plus its kinetic energy as denoted by the classical formula for it.

7. "Ein solcher Raum [der "feldfreie" Minkowski-Raum] ist bezüglich seiner metrischen Eigenschaft dadurch charakterisiert, daß $dx_1^2 + dx_2^2 + dx_3^2$ das Quadrat des mit einem Einheitsmaß gemessenen räumlichen Abstandes zweier infinitesimal benachbarter Punkte eines dreidimensionalen raumartigen Querschnittes ist (Pythagoreischer Satz), während dx_4 der mit geeignetem Zeitmaß gemessene zeitliche Abstand zweier Ereignisse mit gemeinsamen (dx_1, dx_2, dx_3) ist. Dies zusammen kommt -- wie mit Hilfe der Lorentz-Transformationen leicht zu zeigen ist -- darauf hinaus, daß der Größe

$$ds^2 = dx_1^2 + dx_2^2 + dx_3^2 - dx_4^2$$

eine objective metrische Bedeutung zukommt." (1916, p. 99) The distance equation, the "spacetime metric", can also be expressed inversely: $(ds)^2 = (cdt)^2 - [(dx)^2 + (dy)^2 + (dz)^2]$.

The spacetime metric has to be either (-+++) (the former) or (+---) (the alternative) in signature, rather than the (++++) of Euclidean space metric signature, in order to be compatible with Lorentz transformation. Poincaré discovered this spacetime metric several years before Minkowski. (TD, p. 59; also CM: "Poincaré note enfin que la transformation de Lorentz et les transformations associées sont les éléments d'un 'groupe' au sens mathématique du mot (aujourd'hui le groupe de Poincaré, dont celui de Lorentz est un sous-groupe)... Les groupes ont des invariants et Poincaré trouvera l'invariant de son groupe : la quantité $L^2 - c^2 T^2$ où L représente l'intervalle de longueur et T l'intervalle de temps.") Consult Paul J. Nahin's detailed explanation for the necessity of the subtraction of time difference from space distance (PJN, p. 101 - 3): "For cdt', it greatly helps to think carefully about just what dt' is. It is the total differential change in time in the prime system, and it depends on two unprimed variables, x and t, because

$$t' = (t- vx/c^2)/ \sqrt{1 - (v/c)^2}$$

"From calculus we have a result due to Euler (1734)

$$dt' = \frac{\partial t'}{} dx + \frac{\partial t'}{} dt.$$

$$\partial x \qquad \partial t$$

which simply says that the total differential change of t' is the sum of the partial differential changes with respect to x and t, with each such change calculated while holding the other variable fixed. Thus

$$dt' = \frac{-v/c^2}{\sqrt{(1 - (v/c)^2)}} dx + \frac{1}{\sqrt{(1 - (v/c)^2)}} dt$$

Similarly,

$$dx' = \frac{\partial x'}{\partial x} dx + \frac{\partial x'}{\partial t} dt.$$

and, since $x' = (x - vt)/\sqrt{(1-(v/c)^2)}$, then we have

$$dx' = \frac{1}{\sqrt{(1 - (v/c)^2)}} dx - \frac{v}{\sqrt{(1 - (v/c)^2)}} dt$$

From these two results it immediately follows that

$$(dt')^2 = \frac{\frac{v^2}{c^4}(dx)^2 - 2\frac{v}{c^2}(dt)(dx) + (dt)^2}{1-(v/c)^2},$$

$$(dx')^2 = \frac{(dx)^2 - 2v(dt)(dx) + v^2(dt)^2}{1-(v/c)^2}.$$

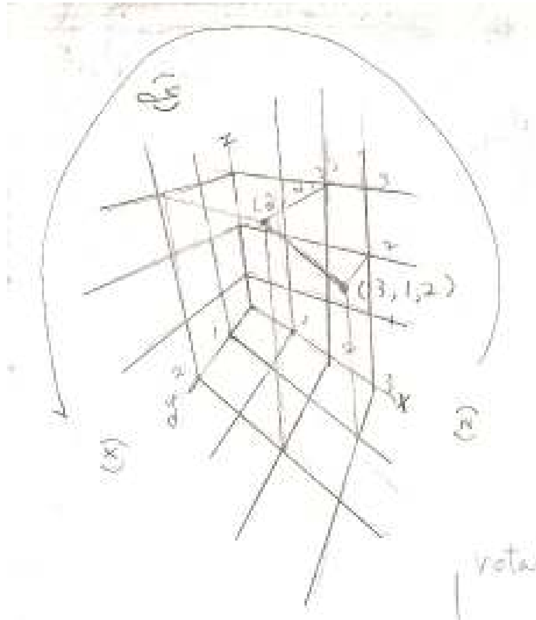
Now, suppose (incorrectly...) that $(ds)^2 = (cdt)^2 + (dx)^2 + (dy)^2 + (dz)^2$ is the spacetime metric.To check this for invariance, I will next calculate $(ds')^2 = (cdt')^2 + (dx')^2 + (dy')^2 + (dz')^2$. This gives

$$(ds')^2 = \frac{\left[\frac{v^2}{c^2}(dx)^2 - 2v(dt)(dx) + c^2(dt)^2\right]}{1-(v/c)^2} + \frac{\left[(dx)^2 - 2v(dt)(dx) + v^2(dt)^2\right]}{1-(v/c)^2} + (dy)^2 + (dz)^2.$$

This expression for $(ds')^2$ is clearly *not* equal to $(ds)^2$." Invariance is established only with $(ds)^2 = -c^2(dt)^2 + (dx)^2 + (dy)^2 + (dz)^2$, with which $(ds)^2 = (ds')^2 = -c^2(dt')^2 + (dx')^2 + (dy')^2 + (dz')^2$.

8.

9. The Euclidean space in the figure below, for example, has undergone a rotation, such that, afterwards, x = z', y = x', and z = y'. Point 1 with its coordinates (3, 1, 2) has after rotation (1, 2, 3). Now $3^2 + 1^2 + 2^2 = 1^2 + 2^2 + 3^2$, and the distance between the two points is invariant with respect to rotation: $(3 - 1)^2 + (1 - 2)^2 + (2 - 3)^2 = (1 - 2)^2 + (2 - 3)^2 + (3 - 1)^2$.



But note that "[u]sing √-1c(dt) = c(√-1dt), using if you will "imaginary time"... gives us the interval invariance we want, but it also makes it pretty clear that time really is fundamentally different from space, a point that many science writers today have muddied in overly simplistic popularization on relativity theory." (PJN, p. 103.) The time dimension in Minkowski's four dimensional world is just not the same thing as the fourth dimension of a four dimensional Euclidean space, as shown in the difference between the respective metric tensors for the Minkowski's world and the Euclidean four dimensional space. At this point however we shall not disturb Einstein's train of thought with this subtle point.

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