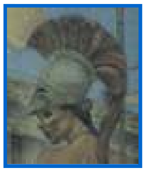


Scientific Enlightenment, Div. Two
A. 4: The Problem of Representation (and the Constitution of Classical Mechanics)
Chapter 7: Fermat's principle of least time

Here we want to demonstrate that Fermat's principle of least time is at bottom the principle of conservation. The original argument which Fermat used to demonstrate this principle of least time is geometrical and highly esoteric at that (c. f. Dugas, *A History of Mechanics*, p. 254 - 259) but seems to reveal, as the underlying *Leitmotiv*, a deliberate expansion on his part of the law of inertia into a broader principle of least time ("In his investigation of the refraction of light, Fermat starts 'from the principle, so common and so well-established, that Nature always acts in the shortest way.'" Ibid., p. 254); and insofar as the law of inertia was understood by the earlier founders of classical mechanics as the expanded version of the principle of conservation, the statement of the identity between the principle of conservation and that of least time probably would have appeared self-evident to Fermat himself. But the modern and (from the historical standpoint) anachronistic formulation of the principle of least time through calculus has lost this origin in the intuition of conservation, but can easily regain it also, as will be done below.

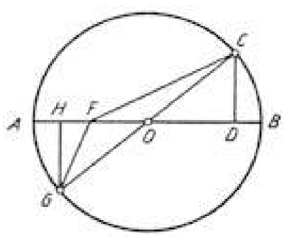


Fig. 89

In other words, Fermat's contribution in this matter is the replacement of least distance of traveling due to conservation (propagation in straight line) with least time of traveling due to conservation: the meaning of the "expansion" of the law of inertia into a more "general" principle. As the issue originally arose with respect to the refraction of light (figure left): "and because... it was not sufficient to have found a point such as F through which the natural motion was accomplished more quickly, more easily and in less time than by the straight line COG, it was also necessary to find the point which allowed the passage from one side to the other in less time than any other there might be." (Cited by Dugas, ibid. p. 255.) The replacement is necessary because Fermat must have realized that "the total amount (of *substantia in motu*) to be conserved" in the law of inertia is only in restricted cases distance traveled but is in general cases actually distance traveled divided by velocity: i.e. time traveled. That is, the conservation of distance traveled (propagation in straight line) is a derivative case of the conservation of time traveled, i.e. *when the velocity stays constant*. As soon as the velocity alters as when light goes into a denser medium which slows it down, it is no longer the distance traveled that is conserved, but the composite distance traveled/ velocity. This also becomes evident in the case of the reflection of light. But at the same time that the real amount conserved is revealed to be not simply distance but rather distance/velocity, distance itself is revealed in these two cases (reflection and refraction) to be the composite of two distances, the distance paralleled to the reflecting or refracting surface divided by the distance light has actually traveled (i.e. the sine function of the angle of incidence, reflection or refraction). A line of thinking alternative to the debate over "living force" (is the "total amount" necessarily conserved mv or mv^2?) is thus being developed here: there are now two different paths of the formulation of the conservation of *substantia in motu*: in terms of energy/momentum and in terms of distance/time.

At the time the greatest obstacle Fermat encountered when he put forward the principle of least time (e.g. *Synthesis ad refractiones*, 1662) was however the prevalent Cartesian consensus (in Fermat's own words) that "the passage of light is easier in dense bodies than in rare ones, which is clearly false" as we know today. (Ibid., p. 254) The Cartesians consequently objected to Fermat's replacement of least distance with least time, e.g. Clerselier's letter to Fermat, May 6, 1662: "[The straight line] is the only thing that Nature tends to in all her motions... The shortness of the time? Never... For, as Nature is determinate in everything she does, she will only and always tend to conduct her works in a straight line." (Cited by Dugas, ibid., p. 258)

In the following, we will examine, first, the principle of least time itself from which the law of inertia (propagation in straight line) is a derivative; then, the case of reflection, and then of refraction (Snell's law), both as determined by the principle of least time, and therefore as consequences of conservation; and, finally, *the analogy between Snell's law and Kepler's second law ("equal area in equal time")* which reveals the true nature of Snell's law as "equal distance in equal time" and so how the law of conservation may manifest itself in such multitude of different laws, but also how consciousness evolves.

(1) The principle of least time itself. Marakani Srikant's (2000) formulation is: "Suppose the light ray starts from point A at time t_A and arrives at point B at time t_B along some path x(t). Fermat's principle states that the trajectory is such that the total time taken for the light ray is the least possible time that is available for any trajectory." This means a straight line. Mathematical representation of "the trajectory minimizes the total time T" can be:

$$T = \int_{t_A}^{t_B} dt$$

i.e. sum of all the instantaneous moments dt of the total time travelled between t_A and t_B. This expressed in terms of space (distance) is:

$$= \int_{x_A}^{x_B} \frac{dt}{dx}\, dx$$

We are trying to establish the composition of time traveled: T = distance traveled/ velocity, whose infinitesimal component dt is therefore: dx/(dx/dt) = (dt/dx)dx = dt. Thus T = the sum of all products *of* each infinitesimal time (dt) divided by each infinitesimal distance (dx) *and* infinitesimal distance between point A and B. "Least time" then means $\delta T = 0$. Srikant's formulation of Fermat's principle of least time in terms of $\delta T = 0$ is from the hindsight of quantum mechanics: "the precise statement of Fermat's principle is not that it minimizes the time taken to go from A to B, but rather that the path be such that the first order variation in T [time taken] between neighboring paths be zero. In most cases this yields the principle of least time, but is a more general statement."

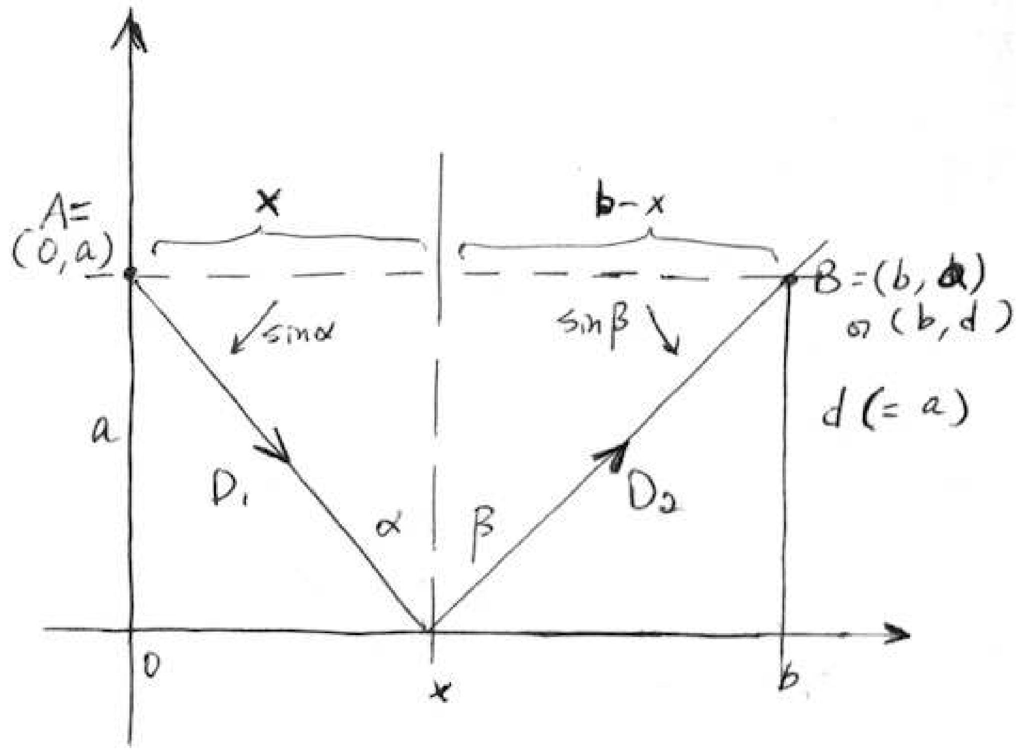
Classically speaking, however, since each infinitesimal portion of T is (dt/dx)dx (inverse of velocity x distance) which is the "total amount" of the propagation of light across distance dx, this amount has to be conserved or remain the same for the next interval dx or dt, and the next and so on, because nothing more can come out of (or nothing less can disappear from) what is there. Thus insofar as the velocity does not change, the distance dx and time dt cannot vary, so that the total time has to remain the minimal, i.e. the addition of all and the same dt between t_A and t_B -- thus resulting in a path that is a straight line. The law of conservation is first reformulated as the law of inertia, and now the law of inertia is reformulated as the law of least time.

Srikant explains:

> **Scientific Metaphor and Fermat's Principle**
> Since Fermat's principle states that light takes the path of least time, one may wonder how does light "choose" the path of least time? How does it "know" that the other paths take a longer amount of time? If one thinks in a rigid classical manner, then clearly there is no answer since light can only follow *one* path, and hence there is no range of paths from which light could "choose" the correct path.
> A metaphoric approach to Fermat's principle is to think of light being able to take *all possible paths* from point A to point B and then "choosing" the path of least time. In our study of quantum mechanics, we will find that this is precisely what light does, and in fact Fermat's principle is the simplest formulation of a more general principle, called the Principle of Least Action, briefly discussed in Section 11.11.

This means that there is a relationship between the straight line path of a photon produced by the summation of all possible pathways as the most likely path (Feynman) and the conservational principle. As we have seen (Zeno's paradoxes), in quantum mechanics, in order for space-time and motion to be logically consistent, the components of the conjugate pairs (position/ momentum and energy/ time) can no longer be defined precisely together; hence the "quantization" of nature's otherwise continuity also means that the conservation of the total, same amount has shifted from a difference of zero (as in $\delta T = 0$ from t_1 to t_2) to a difference of quantized amount of uncertainty (the energy per time of a particle therefore can vary by $\Delta E \Delta t$ > or = $h/2\pi$ rather than = 0). This is how, on the microscopic level, fuzziness and non-linearity may appear.

(2) Reflection. In the previous case, the linear (straight line) propagation of light is due to the conservation of the quantity distance/velocity (T) whose infinitesimal portion is (dt/dx)dx. It is known that in the case of light's reflection the property "the angle of incidence is equal to the angle of reflection" is a consequence of the principle of least time. Alexander Hahn's explanation works like this (*Calculus*, p. 272 - 3).



"Suppose that a light ray proceeds from some source, strikes a mirror, and is reflected. Let A be a point on the ray before it strikes the mirror and let B be a point on the ray after the reflection. See Figure [above]. Suppose that the surrounding medium is air or a vacuum and let v be the speed of light in this medium. The light ray determines a plane that is perpendicular to the mirror. Place a coordinate system in this plane in such a way that the x-axis runs along the mirror's surface and the y-axis goes through A. Let A = (0,a), B = (b,d) [for simplicity, in my figure d = a], and suppose that the ray reflects off the mirror at x. Let α be the angle of incidence and β the angle of reflection. Let D_1 be the distance from A to x and let t_1 be the time it takes for the ray to travel this distance. Similarly, let D_2 be the distance from x to B and let t_2 be the time of travel for this distance. Observe that $D_1 = vt_1$, and hence $t_1 = D_1/v$. In the same way, $t_2 = D_2/v$. By applying the Pythagorean theorem twice, we obtain

$$D_1 = \sqrt{x^2 + a^2} = (x^2 + a^2)^{1/2} \text{ and}$$

$$D_2 = \sqrt{d^2 + (b - x)^2} = (d^2 + (b - x)^2)^{1/2}$$

Therefore, the time t that it takes for the ray to travel from A to B is

$$t = t_1 + t_2 = D_1 + D_2$$

v v

$$= \frac{1}{v}(x^2 + a^2)^{1/2} + \frac{1}{v}(d^2 + (b - x)^2)^{1/2}.$$

So the time of travel is determined by the function

$$t(x) = \frac{1}{v}(x^2 + a^2)^{1/2} + \frac{1}{v}(d^2 + (b - x)^2)^{1/2}$$

where x is the point of incidence. We can now ask: For which x is the travel time t(x) a minimum? To solve this problem, we will analyze the derivative t'(x) [*which is the infinitesimal portion of T*]. By the chain rule,

$$t'(x) = \frac{1}{2v}(x^2 + a^2)^{-1/2}(2x) + \frac{1}{2v}(d^2 + (b - x)^2)^{-1/2}2(b - x)(-1)$$

$$= \frac{x}{v(x^2 + a^2)^{1/2}} - \frac{b - x}{v(d^2 + (b - x)^2)^{1/2}}$$

What are the critical numbers of t(x)? Note that t'(x) is not defined when $(x^2 + a^2)^{1/2} = 0$ or $(d^2 + (b - x)^2)^{1/2} = 0$.

In the first case, a must be 0. This means that A is on the x-axis and hence on the mirror. But this is not the case. If $(d^2 + (b - x)^2)^{1/2}$ is zero, then d must be 0, and then B is on the mirror. Again, this is not the case. It follows that the only critical numbers are those x for which t'(x) = 0. Setting t'(x) = 0, we get

$$\frac{x}{v(x^2 + a^2)^{1/2}} = \frac{b - x}{v(d^2 + (b - x)^2)^{1/2}}$$

From a look at Figure... we see that

$$\sin\alpha = \frac{x}{D_1} = \frac{x}{(x^2 + a^2)^{1/2}}$$

and

$$\sin\beta = \frac{b - x}{D_2} = \frac{b - x}{(d^2 + (b - x)^2)^{1/2}}$$

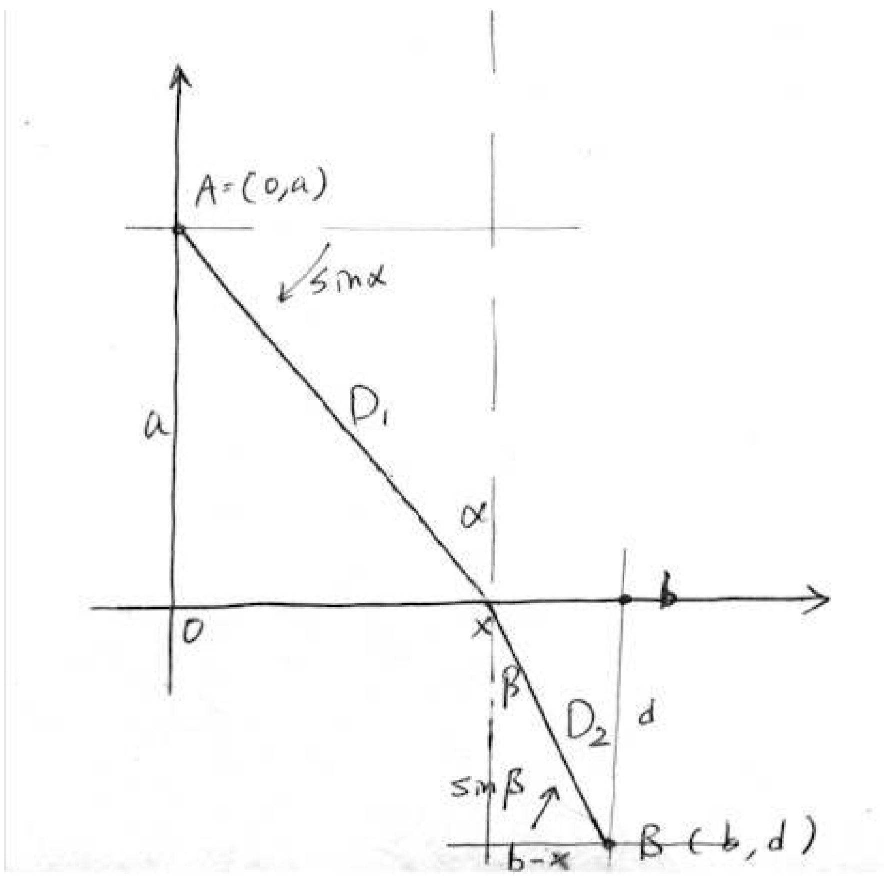
It follows directly that

$$\frac{\sin\alpha}{v} = \frac{\sin\beta}{v.}$$

Multiplying through by v gives us $\sin\alpha = \sin\beta$. Because both α and β are between 0 and 90 degree, it follows that $\alpha = \beta$."

But there is a simpler way to derive this property from the principle of least time and so from *conservation*. $\sin\alpha/v = \sin\beta/v$, clearly an instance of conservation, means $(x/D_1)(1/v) = ((b - x)/D_2)(1/v)$. In other words, the quantity of distance/velocity before and after reflection remains the same -- is conserved. The difference here is that now distance is not the simple distance traveled D_1 or D_2 but the composite x/D_1 or $(b - x)/D_2$ (which is the sine of either the angle of incidence or of the angle of reflection). "Distance" here acquires a composite character because, during light's linear propagation, some flat obstacle comes into its way which is *not perpendicular to its linear path of travel*: the "total amount of distance traveled" now has to be computed through the property of a right triangle. The reflective property of light ($\alpha = \beta$) is really just a round-about way of saying that $(dt/dx)dx$ before and after reflection has to remain the same -- dx is however the derivative of (x/D_1) or $((b - x)/D_2)$; i.e. this "quantity of light's propagation" (distance/velocity) is conserved before and after, $\delta T = 0$ before and after light hits the obstacle. Since the medium is the same, v is constant, so that $x/D_1 = (b - x)/D_2$.

(3) Refraction: Hahn's explanation again. (p. 273 - 5) "Next, consider two homogeneous transparent mediums of different densities. For example, let one of them be air and the other a certain type of glass. Suppose that the boundary between them is a plane. Let A be a point in one medium and B a point in the other, say the denser, medium. Suppose that neither A nor B lies on the boundary separating the two mediums. The path of a light ray traveling from A to B lies in a plane perpendicular to the boundary. At the boundary the ray bends as shown in figure [below]. Such a change in direction is known as *refraction*. Let v_A be the speed of light in the medium containing A and let v_B be the speed of light in the medium containing B. The angle α is the angle of incidence, and the angle β is the angle of refraction."



"We saw that in the case of a reflected ray, there is a connection between the relevant angles (the angles of incidence and reflection are equal). Is there also a connection between the angles α and β in the case of a refracted ray? There are many ways to connect A and B with two line segments that meet at the boundary. Of all these possibilities, which one will a light ray pick out? We will see that Fermat's principle provides the answer to both of these questions."

"Consider the plane determined by the light ray and place a coordinate system so that the x-axis is on the boundary and the y-axis goes through A... Let A = (0, a) and B = (b, d), and suppose that the light crosses the boundary at x. Let D_1 be the distance from A to x and let t_1 be the time it takes for the ray to travel this distance. Similarly, let D_2 be the distance from x to B and let t_2 be the time of travel through this distance. Note that $D_1 = v_A t_1$, so $t_1 = D_1/v_A$. In the same way, $t_2 = D_2/v_B$. By the Pythagorean theorem,

$$D_1 = \sqrt{x^2 + a^2} = (x^2 + a^2)^{1/2} \text{ and}$$

$$D_2 = \sqrt{d^2 + (b - x)^2} = (d^2 + (b - x)^2)^{1/2}.$$

So the time it takes for the ray to travel from A to B is

$$t = t_1 + t_2 = \frac{D_1}{v_A} + \frac{D_2}{v_B}$$

$$= \frac{1}{v_A}(x^2 + a^2)^{1/2} + \frac{1}{v_B}(d^2 + (b - x)^2)^{1/2}."$$

We will see that if time traveled (distance/velocity) is the amount to be conserved ((dt/dx)dx is the amount constantly to remain the same from one interval of travel to the next) then $t_1 = D_1/v_A = t_2 = D_2/v_B$.

"For which x is t a minimum? The calculations are exactly the same as in the earlier situation of reflection:

$$t'(x) = \frac{1}{2v_A}(x^2 + a^2)^{-1/2}(2x) + \frac{1}{2v_B}(d^2 + (b - x)^2)^{-1/2}2(b - x)(-1)$$

$$= \frac{x}{v_A(x^2 + a^2)^{1/2}} - \frac{b - x}{v_B(d^2 + (b - x)^2)^{1/2}}$$

"As before, the only critical points are those where t'(x) = 0. Setting t'(x) = 0 [i.e. the amount (dt/dx)dx is to be conserved, allows no variation, that is, $\delta T = 0$] gives us

$$\frac{x}{v_A(x^2 + a^2)^{1/2}} = \frac{b - x}{v_B(d^2 + (b - x)^2)^{1/2}}$$

"A look at Figure [above] shows that

$$\sin\alpha = \frac{x}{D_1} = \frac{x}{(x^2 + a^2)^{1/2}} \text{ and}$$

$$\sin\beta = \frac{b - x}{D_2} = \frac{b - x}{(d^2 + (b - x)^2)^{1/2}},$$

"and it follows that

$$\frac{\sin\alpha}{v_A} = \frac{\sin\beta}{v_B}."$$

This is the Snell's law of refraction.

Now earlier we learned that distance of light's travel becomes really a composite as soon as an obstacle not perpendicular to its path comes into its way, x/D_1 and $(b - x)/D_2$, that is $\sin\alpha$ and $\sin\beta$. So the conservation of (dt/dx)dx (the infinitesimal portion of light's traveling time) means that, in passing through a boundary between one medium and another, $\sin\alpha/v_A = \sin\beta/v_B$.

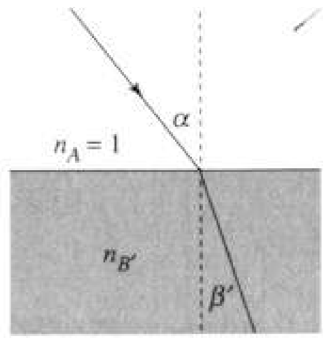
Now, "[i]n air, water, glass, in fact in any translucent medium, light travels more slowly than the c = 186,272 miles per second at which it travels in a vacuum. The *index of refraction n* of any medium is defined to be n = c/v, where v is the speed of light in that medium. The index of refraction of a vacuum is n = c/c = 1. There is no medium in which light propagates faster than it does in a vacuum. So the index of refraction of any medium is n > or = 1... Think of the index of refraction as a measure of the density of the medium: the denser the medium, the less the speed v at which light will travel through it, and hence the higher its index of refraction n." (Hahns, ibid., p. 275) The following table of index of refraction is adopted from Srikant, ibid.

Medium	n = c/cm
Vacuum	1
Air	1.0003
Water	1.33
Plexiglas	1.51
Crown glass	1.52
Quartz	1.54
Diamond	2.42

A single medium can be considered as composed of infinite number of boundaries at all different possible angles relative to the light's path of traveling (from being perpendicular to light's path to lying completely coinciding with it) through which the light passes, but because the index of refraction remains the same -- since it is the same medium -- meaning that the velocity of light stays constant, the angle of refraction is the same as the angle of incidence, so that light is seen as going in a straight line: the principle of least distance or law of inertia is really just a derivative case of the principle of least time *as manifested in the case of refraction*. When the velocity of light changes as it goes into a different medium, however, i.e. when v_A is not equal to v_B, $\sin\alpha$ becomes different, defines a different angle, than $\sin\beta$ does in order to conserve the same amount (dt/dx)dx.

The Snell's law can be converted: since $v_A = c/n_A$ and $v_B = c/n_B$, $\sin\alpha/v_A = \sin\beta/v_B = \sin\alpha/(c/n_A) = \sin\beta/(c/n_B)$ which means $\sin\alpha n_A/c = \sin\beta n_B/c$, which means $n_A\sin\alpha = n_B\sin\beta$.

"Suppose a light ray travels from a vacuum into a denser medium with an angle of incidence α. Now $n_A = 1$, and therefore $\sin\alpha = n_B\sin\beta$. Since $n_B > 1$, it follows that $\sin\alpha > \sin\beta$... and hence $\alpha > \beta$.... The denser medium, shown in Figure [below], produces the smaller angle of refraction, that is, it bends the light ray more" as a matter of conservation of the time for the propagation of light.



(b)

(4) This law of reflection and refraction is in essence very analogous to Kepler's second law of "equal area in equal time". Recall that "[i]n 1609 [Johannes Kepler] published his first two laws of planetary motion in the book *Astronomia Nova*. (1) Each planet P moves in an elliptical orbit with the Sun S at one of the focal points of the ellipse. (See Figure [below].)" (Hahn, ibid., p. 74) This realization came after so much effort at studying the orbit of Mars and yet persistent failure to fit the data concerning this orbit with the ideal conception of the planetary orbit as a circle which his sense of aesthetic, i.e. his conception of the order of the cosmos based on the more compact and restricted experiential horizon, dictated. The concession to the less ideal elliptical orbit due to the demand by more and new empirical data constituted another stepping stone in the process of breaking through the *cosmos* of the functional perspective to arrive at the *solar system* and finally the *Universe* of the structural perspective which was based on more differentiated, complex, and larger experiential horizon. "(2) A given planet sweeps out equal areas in equal time." (Ibid.) The intuition of conservation was dictating the formation of a new "numerology".

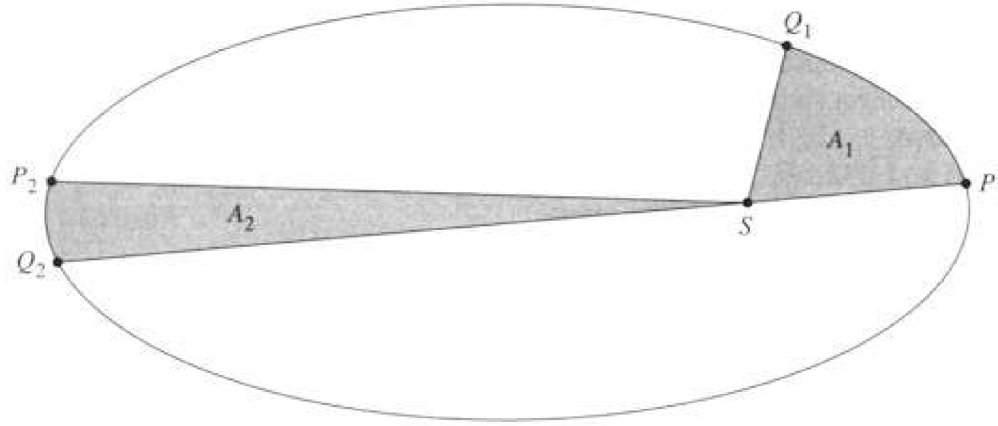


Figure 4.3

(Taken from Hahn, ibid., p. 75.)

"Stated more precisely, this second law says the following: Suppose that it takes a planet time t_1 to move from position P_1 to Q_1 in its orbit, and that it traces out the area A_1 in the process. See Figure [above]. Suppose that it later moves from P_2 to Q_2 in time t_2 while tracing out the area A_2. According to Kepler's second law, if $t_1 = t_2$, then $A_1 = A_2$. A moment's thought shows that the second law implies that a planet moves faster when it is closer to the sun than when it is farther away. The second law also implies that if $t_1 = t_2 + t_2$, then $A_1 = A_2 + A_2$. So if $t_1 = 2t_2$, then $A_1 = 2A_2$... In general, if $t_1 = kt_2$ for some positive constant k, then $A_1 = kA_2$. Now let t_1 and t_2 be any two time intervals and let $t_1/t_2 = k$. So $t_1 = kt_2$, and, as just asserted, $A_1 = kA_2$. So $A_1/t_1 = kA_2/kt_2 = A_2/t_2$. Therefore Kepler's law can be reformulated as follows: Let t be any time interval and let A_t be the area traced out by the planet during time t. Then the ratio A_t/t is the same constant no matter what time t is taken and no matter where in the orbit the motion occurs." (Ibid., p. 74)

The case of the elliptical orbit of the planet is the same as the case of the refraction of light. When the planet orbits around the sun it also has to conserve the amount of time it travels, and time T = distance/ velocity. But here again distance is not simply the distance traveled around the orbit, but the composite (product) of this and the distance away from the sun, i.e. *area*. $T = d/v$ really means $T = A/v$, so that $v = A/T$. Now the infinitesimal portion of time

$$\frac{d(A/v)}{dt} = \frac{dA}{dA/dt} = dA \frac{dt}{dA} = dt$$

is the amount that has to be conserved, i.e. remain constant, hence "equal area in equal time" ($t_1 = t_2 = A_1 = A_2$; or $A_1/t_1 = kA_2/kt_2 = A_2/t_2$). But because the "area traveled" (the *real* "distance traveled") is a composite, the area of the (roughly speaking) triangle, e.g. SP_2Q_2 and yet the speed of the planet is calculated solely as the one-dimensional distance P_2Q_2 divided by time traveled between these two points, when the other dimension, the distance between the planet and the Sun, increases, this speed must appear to decrease in order to conserve the same $T = A/v$ or $v = A/T$, that is, the increment $d(A/v)/dt$ never changes from one infinitesimal portion of the planetary traveling to the next.

The analogy between Fermat's principle of least time (or Snell's law of refraction) and Kepler's second law becomes evident when this Snell's law is reformulated as "equal distance in equal time." Light has to travel this much distance y during a definite time interval, hence when its velocity changes as it goes into a different medium, the angle at which it travels relative to the boundary between the two mediums has to be altered also in order to conserve the same distance y during the same time interval. To use the above example: if $n_A = 1$ and $n_B = 1.3$ (water) and $\alpha = 45$ degree, then $\sin\alpha = n_B \sin\beta = 0.7071 = 1.3(\sin\beta)$; $\sin\beta = 0.543923$ and $\beta = 33$ degree. In medium A (vacuum), in 1 second, at 45 degree angle relative to the flat surface of the water, light travels a distance

$$y = \sqrt{186{,}272^2 - x^2}$$

(since, that is, $x^2 + y^2 = 186{,}272^2$); now in medium B (water), the speed of light is: $1.3 = 186{,}272/v$, $v = 143{,}286$; so that here, in 1 second, at 33 degree angle relative to the flat surface of the water, light travels a distance

$$y' = \sqrt{143{,}286^2 - x'^2}$$

Now it must be the case that

$$\sqrt{186{,}272^2 - x^2} = \sqrt{143{,}286^2 - x'^2}$$

namely, $y = y'$, or "equal distance in equal time".

Secondly, in the case of Kepler's law, it can be seen that the second law is the more general case of which the perfect circular orbit with the planet traveling at a constant velocity is a derivative. This is because the circle is a derivative of the ellipse, i.e. it really is just an ellipse but with zero eccentricity.

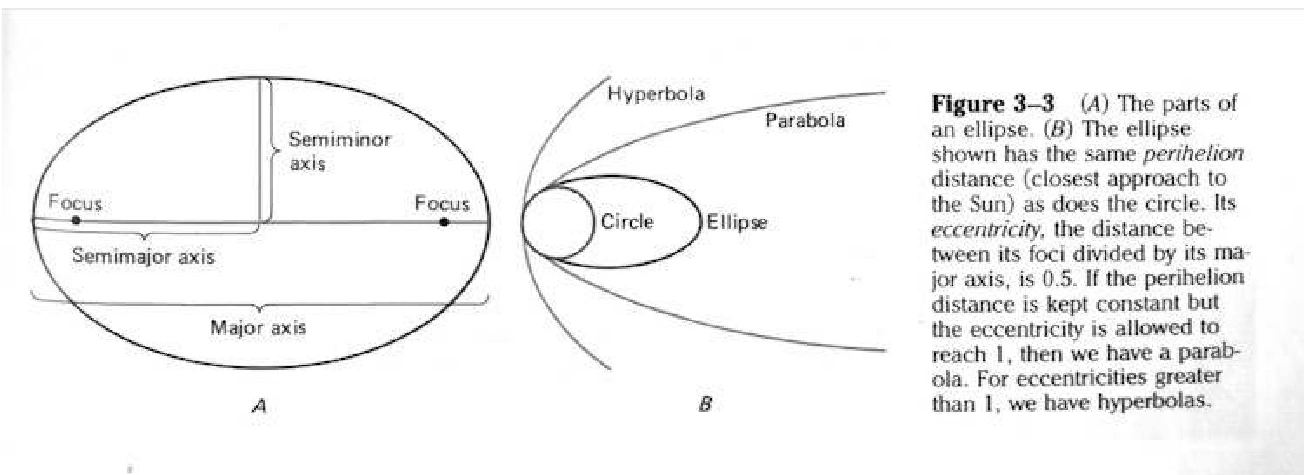
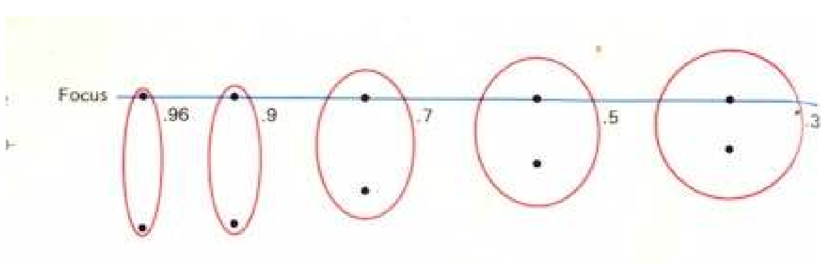


Figure 3–3 (*A*) The parts of an ellipse. (*B*) The ellipse shown has the same *perihelion* distance (closest approach to the Sun) as does the circle. Its *eccentricity*, the distance between its foci divided by its major axis, is 0.5. If the perihelion distance is kept constant but the eccentricity is allowed to reach 1, then we have a parabola. For eccentricities greater than 1, we have hyperbolas.



(Taken from Pasachoff, *Astronomy*, 4th ed., p. 31 - 2: "A series of ellipses of the same major axis but different eccentricities. The *foci* are marked;

> these are the two points inside with the property that the sum of the distance from any point on the circumference to the foci is constant [i.e. conserved; the definition of ellipse]. As the eccentricity -- distance between the foci divided by the major axis -- approaches 1, the ellipse approaches a straight line. As the eccentricity approaches zero, the foci come closer and closer together. A circle is an ellipse of zero eccentricity.")

If the orbit of the planet were a perfect circle, then "equal area in equal time" would simply mean that the planet orbits around the Sun at a constant velocity because the distance between the two remains constant. Thus just as Fermat has discovered that the law of inertia (the principle of least distance, and so of straight line) is simply a restricted case of "Snell's law of refraction", so Kepler has discovered that the circular orbit of constant velocity is only a partial manifestation of the elliptical orbit of "equal area in equal time"; this is in the same manner in which Einstein discovers that Newton's law of gravity is only a partial, derivative case of general relativity ("Newtons Theorie als erste Naeherung", in his words; "Die Grundlage der allgemeinen Relativitaetstheorie", in *The Collected Papers of Albert Einstein*, vol. 6, Princeton Uni. Pr., 1996). This illustrates the evolution of consciousness: as it enlarges its experiential horizon, it notices that what it formerly considers to be the *whole* truth is really only a partial manifestation, a restricted case, or an effect of some larger picture beneath or behind.

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