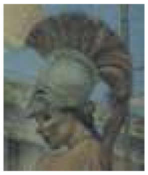


Scientific Enlightenment, Div. One

Book 2: Human Enlightenment of the First Axial

2.B.1. A genealogy of the philosophic enlightenment in classical Greece

Chapter 9: Zeno's paradoxes of motion (and the logical hints at the new physics)

ACADEMY | previous section | Table of Content | next section | **GALLERY**

In *Quantum Theory* (1951) David Bohm has an enlightening discussion about the shattering of the world view of classical mechanics by quantum mechanics.

First, discontinuity (quanta) vs. continuity. Quantum mechanics seems to contradict everyday common sense and classical mechanics because it presents the reality as discrete or segmented into discontinuities rather than as a continuous transition such as we expect from everyday experience and classical mechanics. But in fact if we consider our "common sense" (and so the presuppositions of classical mechanics) carefully, we shall notice that it is self-contradictory and illogical in its assumption of continuity, and the discontinuous world of quantum reality should be expected as logical and consistent. The illogicality of classical mechanics (and so common sense) in respect to the continuous nature of reality manifests itself in its treatment of the relationship between position changed and time elapsed of a moving object. "Our simplest ideas about the position of an object seem to imply that an object with definite position is not moving. That is, if we try to picture the position of an object with *perfect* accuracy, we seem to imagine an object that is at one fixed position and at no other." (Ibid., p. 145) We then construct "motion" out of such notion of position "as a succession of objects at slightly different positions". Note that such construing of motion cannot imply continuous motion, "the idea that a real moving object is *continuously* covering space as time passes." (Ibid.) But the way we in classical mechanics calculate velocity (i.e. through derivative: an object covering an interval of space during some interval of time; distance travelled divided by time elapsed) implies precisely that we are imagining the "fixed" position of a non-moving object together with its motion as the succession of such different but fixed positions: that is, the object is conceived of as resting and moving at the same time. "We may reduce the element of time [elapsed] to a very small value and thus reduce the indefiniteness of position to a correspondingly small value. But we cannot reduce the indefiniteness to zero and still obtain a picture of a moving object, for in picturing an object at an absolutely definite point in space we cannot seem to help picturing it as fixed." The violation of the law of non-contradiction (the combination of definite position and continuous motion leads to the imagery that the object is both at rest and moving) inherent in the differential calculation of classical mechanics prompts Bohm to conclude that "we cannot think of the [non-indefinite] position of an object and of its [continuous] velocity simultaneously." (Ibid.)

Bohm's statement strikes at the heart of the fallacy in the differential equation (of the derivative), such as that for the velocity of a particle at instant t:

$$v(t) = \lim_{\Delta t \to 0} \frac{p(t + \Delta t) - p(t)}{\Delta t} = \frac{p\Delta t}{\Delta t} = \frac{\Delta x}{\Delta t}$$

If the motion is continuous, then the object is changing its position constantly and so has really an indefinite position at time t; but then the essence of differential equation is precisely the presupposition of the possibility of the reduction of position x to a *fixed*, i.e. definite point when the indefiniteness of time Δt is reduced to 0. Something has to give in order to preserve logical consistency, to conform to the law of non-contradiction. We must admit either that velocity and position cannot be made totally definite (through the reduction of indefiniteness to 0) simultaneously or that *motion is in fact quantized*. In both solutions we move into quantum mechanics. First, to have our common sense be logically consistent, we have to imagine a moving particle as not having a definite position at time t. "According to quantum theory, momentum, and hence velocity, can be given an exact meaning only when there is provision for a wave-like structure in space. When this provision is made, the wave packet is in a state of transition through space, in which its average position moves from one point to the next at a fairly definite velocity. But the motion of the wave packet is analogous to our simple picture of a particle in motion [our logically consistent imagery of the moving particle] because, in both, the particle [= wave packet] is thought of as covering *a range* of positions at any instant, while the average position changes uniformly with time. Quantum theory, therefore, gives a picture of the process of motion that is considerably closer to our simplest [i.e. logically consistent common sense] concepts [of *continuous* motion] than does classical theory." (Ibid., p. 146; emphasis added.) If we insist, in our common sense, on imagining the particle at a fixed position, then it either does not move at all or its velocity can no longer be imagined *definitely*: this impossibility of thinking definite position and velocity at the same time is clearly expressed by the uncertainty relation $\Delta x \Delta p$ > or = $h/2\pi$. Quantum mechanics, rather than classical mechanics (as in the derivative), is the logical formulation of our common sense. But not totally, since the uncertainty relations in effect says that no particle can actually be definitively at rest, taking away the validity of our ever imaging it as motionless. "We conclude that our naive pictures [i.e. common sense *logically* formulated] and quantum theory are alike in that they both have the following property: It is possible to give a continuous picture of the motion only if the position is blurred or made indefinite, and it is possible to give a picture of a [moving] particle in a definite position only if we forgo the possibility of picturing it in continuous motion [the second, quantized alternative]." (Ibid., p. 147.)

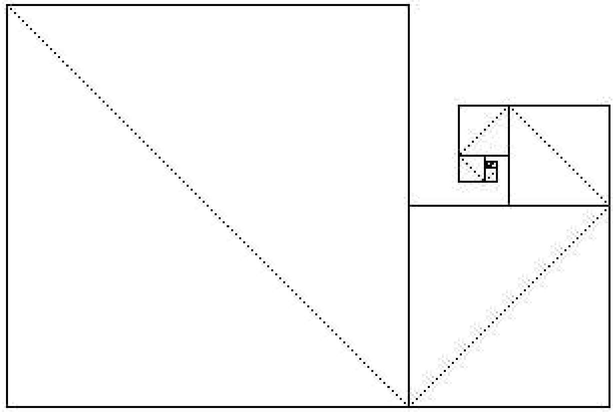
So the common sense of continuous motion, when left to its logical inconsistency with the presence in it of the other common sense of "things having fixed positions", and when formulated thusly quantitatively, turns into the differential equation typical of classical mechanics. When re-formulated logically, it turns into the uncertainty principle governing wave packet (Bohr's type). We can of course remain attached to our common sense experience of fixed positioning and give up continuous motion in favor of quantized. But the common sense of continuous motion seems so ingrained and cannot imaginably be abandoned; but then so is that of fixed positioning. As Bohm points out, the differential equation which formulated in quantitative terms this common sense had this logical self-contradiction inherent in it escape our notice because it seemed to work (in the sense of predicting the results of experiments) in empirical reality. (For example, interior ballistics.) "In other words, the classical idea that a particle has a trajectory for which the derivative can be defined at each point is based only on empirical evidence. Since the days of Newton, the great success of classical theory provided strong empirical evidence and it seemed inevitable and in the nature of things that a continuous trajectory [along with definite positioning of the moving object during motion] was the only conceivable kind that real matter could follow. Yet, on a purely logical basis, there is no reason to choose the concept of a continuous trajectory in preference to that of discontinuous trajectory. It is quite possible that $\Delta x/\Delta t$ shall approach a limit for a while as Δt is made smaller, and then cease to approach a limit as Δt is made smaller still." (Ibid., p. 148.) The experiments leading to quantum theory have shown that, in effect, the classical experiments, dealing only with macroscopic phenomena of motion, are actually imprecise, vague, and partial (restricted to the macroscopic), so that the derivative of differential equation conforms only to incomplete empirical data and so is *not* exactly confirmed thereby. In fact, the break-down of classical mechanics was noticed precisely because the experiments, in going progressively into the microscopic world, were getting more and more precise, i.e. having the empirical data more and more complete, showing the derivative type of thing to be approximate at most and so *empirically invalidating it*. As Bohm points out also, it is precisely Zeno who has demonstrated, in his paradoxes of motion, that the common sense notion of motion must be fundamentally wrong because, logically speaking, motion cannot be possible as continuous and associated with definite positioning. This is the first awakening to the *illogicality* of common sense, everyday experience and the implication of quantum mechanical concepts as the *logical* reformulation, or "repair", of common sense. But, as the close examination of the four paradoxes of motion will show below, not only does motion seem possible only if quantized and indefinite with respect to positioning at an instant of it (one solution to the paradox), but it could also be possible only if space and time form a continuity and be relative (i.e. relativity could in some cases be the alternative solution to quantized nature); but, moreover, *quantized nature seems possible (logically consistent) only given special relativity*.

Leaving their original purpose -- the defense of his teacher Parmenides' vision -- aside for the moment, Zeno's paradoxes consist in two sets, one showing the impossibility of plurality (given continuity in the constitution of things) and the other that of motion. We need only to focus on the latter. The paradoxes for motion are 4, first three showing the impossibility of motion in the common sense or classical mechanics, the last, most complex, seeming to show the implication of special relativity by quantized motion.[1]

1. The stadium, that "it is impossible to move and traverse the stadium" (οτι ουκ ενδεχεται κινεισθαι ουδε το σταδιον διελθειν). "The first is about the non-existence of motion because the moving thing [starting from one side of the stadium] must arrive at half-way before it reaches the end [e.g. the other side of the stadium]..." πρωτος μεν ο περι του μη κινεσθαι δια το προτερον εις το ημισυ δειν αφικεσθαι το φερομενον η προς το τελος.... (From Aristotle, in Kirk and Raven, ibid., p. 292) And before reaching the half way, it must reach half-way of this half-way, *ad infinitum*. "In other words, on the assumption that space is infinitely divisible and that therefore any finite distance contains an infinite number of points, it is impossible to reach the end of an infinite series in a finite time." (Ibid., p. 293)

Of the two presuppositions of motion as conceived in the differential equation of classical mechanics and in common sense, that it is continuous and that the moving object has a definite position at instant t, the first one is shown here to lead to the paradox of infinite division in finite time. Hence motion is impossible, a logically self-contradictory notion, if continuous. If, on the other hand, space and time, and so traverse in space over time elapsed ($\Delta x/\Delta t$) is quantized, so that Δx cannot be reduced indefinitely, then the paradox is resolved.

Aristotle rebukes this paradox thusly: διο και ο Ζηνωνος λογος ψευδος λαμβανει το μη ενδεχεσθαι τα απειρα διελθειν η αψασθαι των απειρων καθ'εκαστον εν πεπερασμενωι χρονωι. διχως γαρ λεγεται και το μηκος και ο χρονος απειρον, και ολως παν το συνεχες, ητοι κατα διαιρεσιν η τοις εσχατοις. των μεν ουν κατα ποσον απειρων ουκ ενδεχεται αψασθαι εν πεπερασμενωι χρονωι, των δε κατα διαιρεσιν ενδεχεται. και γαρ αυτος ο χρονος ουτως απειρος, ωστε εν τωι απειρωι και ουκ εν τωι πεπερασμενωι συμβαινει διιεναι το απειρον, και απτεσθαι των απειρων τοις απειροις, ου τοις πεπερασμενοις. "Hence Zeno's argument is false in assuming that it is impossible to traverse the infinite or reach [lit. touch] the infinite, in each case, in a finite time. For in two ways can length and time, or generally *anything continuous*, be said to be infinite, i.e. either as infinitely divisible or infinite in extreme [i.e. in extension]. So while, with regard to quantitative infinity [i.e. infinite extension], it is impossible to reach this infinite in a finite time, such is possible with regard to infinite divisibility. For [in this respect of divisibility] time itself is infinite [infinitely divisible], so that it is in infinite time and not in finite time that the thing traverses over the infinite [sum of infinitesimal divisions of distance], or that the thing reaches the infinite by infinity [i.e. infinite number of infinitesimal moments], not by finitude." (Kirk and Raven, ibid., p. 293.) This is one way to solve the paradox, by showing that the underlying assumption of the paradox (infinite division in finite time) is wrong. Another way, typical in modern mathematics, is to show that the infinite series of divisions of an interval of space, although infinite, has a finite sum, i.e. that it "converges".[2] "This may be a useful pedagogical device for beginning calculus students, but it misses an interesting and important philosophical point implied by Zeno's arguments. To see this, we can re-formulate the essence of these two arguments in more modern terms, and show that, far from being vitiated by the convergence of infinite series, they actually depend on the convergence of the geometric series... we could construct 'Zeno's maze' by having a beam of light directed around a spiral as shown below:



...the total path is finite [i.e. 'converges'], but has no end, i.e., no final direction, and a ray propagating along this path can neither continue nor escape. Of course, modern readers may feel entitled to disregard this line of reasoning, knowing that matter consists of atoms which are not infinitely divisible, so we could never construct an infinite sequence of geometrically decreasing mirrors. Also, every photon has some finite scattering wavelength and thus cannot be treated as a 'point particle'. However, these arguments merely confirm Zeno's position that the physical world is not scale-invariant or infinitely divisible. Thus, we haven't debunked Zeno, we've merely conceded his point. Of course, this point is not, in itself, paradoxical. It simply indicates that at some level the physical world must be regarded as consisting of finite indivisible entities [*must be quantized*]." (From Zeno and the Paradox of Motion, Kevin Brown.)

2. "Achilles and the tortoise", employs the same reasoning but not with absolute motion but relative motion of two objects. εστι δ'ουτος οτι το βραδυτατον ουδεποτε καταληφθησεται θεον υπο του ταχιστου. εμπροσθεν γαρ αναγκαιον ελθειν το διωκον οθεν ωρμησε το φευγον, ωστ'αει τι προεχειν αναγκαιον το βραδυτερον. εστι δε και ουτος ο αυτος λογος τωι διχοτομειν, διαφερει δ'εν τωι διαιρειν μη διχα το προσλαμβανομενον μεγεθος. "It is this that in a race the slowest [tortoise] can never be overtaken by the quickest [Achilles]. For before that the pursuer must first come to the point where the pursued started, so that the slower [the pursued tortoise] must hold a lead. [By the time Achilles reaches the second starting point of the tortoise, this latter will have again moved on, and so *ad infinitum*.] This argument is the same as that based on bisection, and it differs from the latter only in that the [infinitely] successive lengths are not divided into halves." (Aristotle, *Phys.* Z9, 239 b 14; Kirk and Raven, ibid., p. 294.) Such wise Zeno disproves continuous motion, implying that our perception of motion implies, simply by the necessity of logic(al consistency), the discreteness of space, time, and motion.

The next two paradoxes are meant by Zeno to disprove *discontinuous motion also*, for at the time the Greeks held to two distinct views of the nature of space and time. "Either space and time are infinitely divisible, in which case motion is continuous and smooth-flowing [our classical mechanics]; or else they are made up of indivisible minima -- ατομα μεγεθη -- in which case motion is what Lee aptly calls 'cinematographic', consisting of a succession of minute jerks [quantized]." (Ibid., p. 292) This recalls that "[i]n the 5th century of the current era, there was a bitter argument in India between the Sankhya Hindus and the Buddhists about the nature of Universal Flux. Debates were held which lasted for days, and would attract huge crowds", with the Buddhist holding to the view of flux of time as quantized and the Samkhya seeing it as a continuous movement of fluctuation from the conserved substratum. This reminds, furthermore, of the opposition between Schroedinger's wave mechanics and Heisenberg's matrix. (David Harrison)

3. The flying arrow: ...οτι η οιστος φερομενη εστηκεν. συμβαινει δε παρα το λαμβανειν το χρονον συγκεισθαι εκ των νυν. μη διδομενου γαρ τουτου ουκ εσται ο συλλογισμος. "That the flying arrow is at rest [lit. standing]. This results from the assumption that time is composed of [quantized] moments [lit., 'nows']. If this not be granted, the conclusion will not stand." (Aristotle, *Phys*. Z 9, 239 b 30; ibid., p. 294.) "This third argument can be confidently reconstructed as follows: 'An object is at rest when it occupies a space equal to its

own dimensions. An arrow in flight occupies, at any given moment, a space equal to its own dimensions. Therefore an arrow in flight is at rest.'" (Ibid.)

This paradox, now showing that even quantized motion consists in logical self-contradiction (for the arrow, with discontinuous motion, is at rest and flying at the same time), can be resolved in two ways, one leading to the uncertainty principle and the other to special relativity.

An arrow, flying quantizedly, occupies in each quantized moment of its flight a length x equal to its length, and at next quantized moment it is at next x, skipping altogether the interval or difference between x and x_1 all at once in "its quantum jump". Flying is impossible for there is no difference, at instant t, between the arrow at rest and the arrow in motion *because both have fixed position x*, which is then the presupposition that has to be discarded in order to restore the difference between motion and rest and so to make motion possible again. The problem is therefore the same as before: object in motion cannot have well-defined position x at instant t of its motion. The flying arrow, in order to fly, at instant t of its flight must have indefinite position Δx which here would then comprehend at least x and x_1. That is, the arrow in flight does not, at instant t of its flight, occupy a space equal to its length but longer than its length. The uncertainty principle must be presupposed (that position cannot be defined precisely at the same time as momentum) as a matter of logical consistency in thinking about motion.[3]

In "Zeno and the paradox of motion" (ibid) two more solutions to the paradox are proposed, though we are mainly interested in the second one of relativity: "But if there is literally no physical difference between a moving and a non-moving arrow in any given discrete instant, then how does the arrow *know* from one instant to the next if it is moving? In other words, how is causality transmitted forward in time through a sequence of instants, in each of which motion does not exist?

"It's been noted that Zeno's 'Arrow' argument could also be made in the context of continuous motion, where in any single slice of time there is (presumed to be) no physical difference between a moving and a non-moving arrow. Thus, Zeno suggests that if all time is composed of instants (continuous or discrete), and motion cannot exist in any instant, then motion cannot exist at all. [The first solution is then to suppose the uncertainty principle.]

"[Solution 2: conceding to Zeno that motion is an illusion projected from a higher dimension:] A naive response to this argument is to point out that although the *value* of a function f(t) is constant for a given t, the *function* f(t) may be non-constant at t [something like the uncertainty principle]. But, again, this explanation doesn't really address the phenomenological issue raised by Zeno's argument. A continuous function (as emphasized by Weierstrass) is a *static* completed entity, so by invoking this model we are essentially agreeing with Parmenides that physical motion does not truly exist, and is just an illusion, i.e., 'opinions', arising from our psychological experience of a static unchanging reality.

"Of course, to accomplish this we have expanded our concept of 'the existing world' to include another dimension. If, instead, we insist on adhering to the view of the entire physical world as a purely spatial expanse, existing in and progressing through a sequence of instants, then we again run into the problem of how a quality that exists only over a range of instants can be causally conveyed through any given instant in which it has no form of existence. Before we blithely dismiss this concern as non-sensical, it's worth noting that modern physics has concluded (along with Zeno) that the classical image of space and time was fundamentally wrong, and in fact motion would *not* be possible in a universe constructed according to the classical model. [Hence another presupposition is identified in the self-contradictory common sense of motion -- whether discontinuous or continuous -- whose discard would eliminate the self-contradiction: that space and time are absolute and independent of each other.]

"[So we have solution 3:] The theory of special relativity answers Zeno's concern over the lack of an instantaneous difference between a moving and a non-moving arrow by positing a fundamental re-structuring of the basic way in which space and time fit together, such that there really *is* an instantaneous difference between a moving and a non-moving object, insofar as it makes sense to speak of 'an instant' of a physical system with mutually moving elements. Objects in relative motion have different planes of simultaneity, with all the familiar relativistic consequences, so not only does a moving object look different to the world [*unlike the object at rest*], but the world looks different to a moving object.

"This resolution of the paradox of motion presumably never occurred to Zeno, but it's no exaggeration to say that special relativity vindicates Zeno's skepticism and physical intuition about the nature of motion. He was correct that instantaneous velocity in the context of absolute space and absolute time does not correspond to physical reality, and probably doesn't even make sense. From Zeno's point of view, the classical concept of absolute time was not logically sound, and special relativity (or something like it [and also quantum and the uncertainty principle, perhaps]) is a *logical* necessity, not just an empirical fact. It's even been suggested that if people had taken Zeno's paradoxes more seriously they might have arrived at something like special relativity centuries ago, just on logical grounds. This suggestion goes back at least to Minkowski's famous lecture of 'staircase wit'... Doubtless it's stretching the point to say that Zeno anticipated the theory of special relativity, but it's undeniably true that his misgivings about the logical consistency of motion in its classical form were substantially justified. The universe does not (and arguably, *could* not) work the way people thought it did." The solution is therefore the interdependence of space and time as in relativity.

4. "The moving rows in the stadium" shows amazingly that quantum might *imply* relativity definitively, not just implying the uncertainty principle as alternative solution to the paradox of flying arrow *next to* relativity. (Note the original Greek text by Aristotle is extremely confusing and a lot of interpretative work goes into the translation, for which we follow mostly Gaye in Kirk & Raven.)

τεταρτος δ' ο περι των εν σταδιωι κινουμενων εξ εναντιας ισων ογκων παρ'ισους, των μεν απο τελους του σταδιου των δ'απο μεσου, ισωι ταχει, εν ωι συμβαινειν οιεται ισον ειναι χρονον τωι διπλασιωι τον ημισυν. εστι δ'ο παραλογισμος εν τωι το μεν παρα κινουμενον το δε παρ' ηρεμουν το ισον μεγεθος αξιουν τωι ισωι ταχει τον ισον φερεσθαι χρονον. τουτο δ'εστι ψευδος. οιον εστωσαν οι εστωτες ισοι ογκοι εφ'ων τα ΑΑ, οι δ'εφ'ων τα ΒΒ αρχομενοι απο του μεσου των Α, ισοι τον αριθμον τουτοις οντες και το μεγεθος, οι δ'εφ'ων τα ΓΓ απο του εσχατου, ισοι τον αριθμον οντες τουτοις και το μεγεθος, και ισοταχεις τοις Β. συμβαινει δη το πρωτον Β αμα επι τωι εσχατωι ειναι και το πρωτον Γ, παρ'αλληλα κινουμενων. συμβαινει δε και το Γ παρα παντα τα Β διεξεληλυθεναι, το δε Β παρα τα [Α] ημισυ. ωστε ημισυν ειναι τον χρονον. ισον γαρ εκαστερον εστιν παρ'εκαστον. αμα δε συμβαινει τα Β παρα παντα τα Γ παρεληλυθεναι. αμα γαρ εσται το πρωτον Γ και το πρωτον Β επι τοις εναντιοις εσχατοις, ισον χρονον παρ'εκαστον γινομενον των Β

The fourth is about [two] rows of bodies, each row of an equal [number of] bodies of equal size, moving, in a stadium, from opposite sides [toward and passing each other] with equal velocity, the one row originally occupying the space between the goal and the middle point of the course [i.e. ΓΓΓΓ] and the other that between the middle point and the starting post [BBBB]. This [Zeno] thinks results in the conclusion that half of a given time is equal to double that time. The fallacy [of his reasoning] lies in this, that a body takes an equal time in moving with equal velocity to pass a moving body and resting body all of equal size. This is false. For example [so runs the argument], let AA be stationary bodies of equal size, and BB, starting from the half of the course from the starting post to the middle of the AA, equal in number and size to AA, and ΓΓ, starting [on the other side] from the goal [to the middle of the As], equal in number, size, and velocity to BB. [Then three consequences follow:]
First, the first B reaches the last [Γ] at the same moment as the first Γ [reaches the last B] as the two rows pass each other.
[Second,] while the [first] Γ has passed all the Bs, the [first] B has passed only half [As], so that the time [of first B passing A] is half [of Γ passing B]. For each of the two

οσον περ των Α, ως φησι, δια το αμφοτερα ισον χρονον παρα τα Α γιγνεσθαι.

[rows] takes equal time in passing each body. [The assumption seems to be that *the time for a B to pass an A must be equal to that for a Γ to pass a B, and therefore that for a B to pass a Γ: relativity.*] [Thirdly,] at the same moment all the Bs have passed all the Γs. For the first Γ and the first B will simultaneously reach the opposite ends, for the time for [the first Γ] to pass each of Bs is equal to that [for it] to pass each of As, as [Zeno] says, because the time for both [the first B and the first Γ] to pass all the As is equal. [Aristotle, *Phys.* Z 9, 239 b33. Diagram of Alexander *ap.* Simplicium *Phys.* 1016, 14 is given, with Γ replaced by C, and Δ by D. All from Kirk & Raven, ibid., p. 295-6)

```
        AAAA
D    BBBB ->      E
       <- CCCC
```

- A ογκοι εστωτες (stationary bodies)
- B ογκοι κινουμενοι απο του Δ επι το E (bodies moving from D to E)
- C ογκοι κινουμενοι απο του E επι το Δ (bodies moving from E to D)
- D αρχη του στατδιου (starting post of *stadium*)
- E τελος του σταδιου (goal of *stadium*)

The paradox, just as the flying arrow, attempts to demonstrate the impossibility of motion given the quantized view of space and time. However, this only demonstrates that the quantized view, by the necessity of logical self-consistency, implies a maximal speed of motion, hence special relativity. We can qualify the argument by specifying that A, B, and C represent the minimally possible length, the quantum space, like the Planck's length of 10^{-33} cm (actually 1.6×10^{-33}cm); and the time t for B to pass A or for C to pass A as the minimally possible time, Planck's time of 10^{-43} second. t = n is then an integer multiple of Planck's time and the event described above becomes the succession of these multiples of t. These, along with the smallest mass possible, Planck's mass 2×10^{-5} gram, we recall, are produced out of the combinations of the fundamental constants in physics, G, c, and h-cross, as first pointed out by Max Planck himself in 1913[4]. (Gribbin, *In Search of the Big Bang*, p. 335. Kirk and Raven make the same point, that the ογκοι has to mean indivisible minimum of space and the time of B or C passing A one indivisible minimum of time. Aristotle has certainly failed to grasp the point.)

```
              AAAA
t = 1  BBBB ->
                <- CCCC
              AAAA
t = 2   BBBB
                   CCCC
              AAAA
t = 3    BBBB
                 CCCC
              AAAA
t = 4     BBBB
               CCCC
```

```
              AAAA
t = 5         BBBB
              CCCC
```

"But once so much [the qualification of the minimum length and time] is granted, then the rest of the argument is valid. For while each B has passed two As -- which, by the data, means in two indivisible minima of time [Planck's time x 2] -- each Γ has passed four Bs -- which again by the data must have taken four indivisible minima [Planck's time x 4]. It is true, of course, that unless the argument is concerned with indivisible minima it is, as Aristotle says, totally invalid. But as soon as it is seen to be concerned with indivisible minima, both of space and time, then it does most ingeniously demonstrate that these so-called indivisible minima are divisible after all." (Ibid., p. 297) Please note what is happening here. In 2 t (2 x Planck's time) B has passed 2 As but 4 Γs; so the time it takes to pass 1 Γ is 1/2 t. But t (Planck's time) is by definition indivisible. And yet once the minimal length (Planck's length) is set stationary and another minimal length is set passing by it at the speed of 1 minimal (Planck) length per 1 minimal (Planck) time then self-contradiction results -- as soon as a third Planck length comes passing by at the same speed *from the other end in the opposite direction* -- that the indivisible is again divisible. The only way out is to posit that, in this case, half of the time of minimal (Planck's) length passing another *stationary* minimal (Planck) length at the speed of 1 minimal (Planck) length per 1 minimal (Planck) time is equal to double this time if the stationary minimal (Planck) length is set moving in the opposite direction at the same speed. For Zeno this is self-contradiction, so that motion is impossible, whether reality is quantized or not. *But this is in fact reality and is called special relativity*. That is, in order for nature to remain quantized in terms of Planck's length and Planck's time, there has to be a maximally possible speed, c, or the speed of light, so that B and Γ in the above are travelling at this maximal speed. In fact, "[t]he Planck time is the time it takes light to travel across the Planck length, which is some 10^{-20} times the distance 'across' a proton." (Gribbin, ibid.) Relativity of time then results, so that half of the time in one perspective may be equal to double the time in another. If 1 C passing 1 A = (during) t, and 1 B passing 1 A = t, and if B and C are then passing each other by, the velocity of B from the perspective of C is not

$$v(B) + v(C) = 2v$$

but rather, according to the theorem of the addition of velocities in Einstein's special relativity:

$$v(B) = \frac{(v(B) + v(C))}{(1 + v(B)v(C)/c^2)}$$

And since both v(B) and v(C) = c so the result (the velocity of B to C) is still c, i.e. for C, it itself (C) is like stationary just like A and watching B passing 2 As while passing 4 of itself. Hence half the time = double the time. We recall that the Michelson and Morley experiment of 1887 that arranged light beams and mirrors to measure the speed of light moving in the same direction as the Earth through space and of that in the opposite direction always paradoxically gave the same result. "Common sense tells you that the answers ought to be different... If I ride in a bus traveling at 30 miles per hour and another bus on the opposite side of the highway passes in the opposite direction at 30 miles per hour, relative to me the second bus is moving at 60 miles per hour. But light isn't like that. The earth moves through space at some velocity, which we might call v. A light beam overtaking us at velocity c does *not* have a speed c - v, nor does a beam of light approaching us from the opposite direction have a speed c + v. Whatever our velocity and whichever direction the beam of light is coming from, when we measure its speed we always get the answer c." (John Gribbin, ibid., p. 73.) c is the maximal speed possible in the Universe. Already from James Clerk Maxwell's equations for electromagnetism (1860s) "light emerges... as a constant, the value of which does not depend on the velocity of the observer who is attempting to measure it." ("Special Relativity", *Physics Today*, World Book, Inc., Chicago, p. 138) Hendrik Lorentz' transformation gave further confirmation of this. In this new understanding Einstein derived the above equation for calculating relative velocity in 1905. In everyday circumstances the velocity we experience is so slow (like 30 miles /hour), so v_1v_2/c^2 is virtually 0. Relativity becomes noticeable when the two velocities approach c.

We therefore realize the common sense is illogical and based in fact on empirical reality, which has misled us because our experience of it is incomplete (imprecise, macroscopic, and at low velocity).

To maintain h-across, then, we need c. That is to say, logical consistency, the conformity to the law of non-contradiction, means that motion is necessarily quantized (by h) and that, this being so, motion quantized in the minimal by h necessarily implies that it is also bound in the maximal by c. Quantum mechanics implies relativity as a matter of logical consistency: this is the lesson of Zeno's paradox.[5]

The author of "Zeno and the Paradox of Motion" concludes, after the fourth paradox: "Surely we can forgive Zeno for not seeing that his argument can only be satisfactorily answered -- from the standpoint of physics -- by assuming Lorentzian invariance and the relativity of space and time. According to this view, with its rejection of absolute simultaneity, we're inevitably led from a dynamical model in which a single slice of space progresses 'evenly and equally' through time, to a purely static representation in which the entire history of each worldline already exists as a completed entity in the plenum of spacetime. This static representation, according to which our perceptions of change and motion are simply the product of our advancing awareness, is strikingly harmonious with the teachings of Parmenides, whose intelligibility Zeno's arguments were designed to defend." This is only to be accepted preliminarily, for Parmenides' Being does not refer to empirical reality -- but not to say that it has no implication for empirical reality. Zeno's argument, moreover, has actually gone far further than he thought, as we shall see presently.

Note that in the present case logic -- the tautology of A = A -- manifested as the law of non-contradiction here leads us from the perception of motion to both h (together with the uncertainty principle) and c: quantum mechanics and relativity. Elsewhere this law of tautology, manifested as the law of Conservation, leads the philosophers (the Buddhists especially) to realize that existence is an illusion, that nothing *really* exists. The fundamental structure of the Universe is logical tautology. Anyone who thinks that quantum mechanics and relativity disprove logic is mistaken: the reality is the opposite. Since both the law of non-contradiction and of Conservation are manifestations of logical tautology, can that quantum mechanics and relativity -- and all the laws of nature we find -- be just aspects of the law of Conservation, that A = A eternally and immutably?

For the meantime, we shall only hold onto the immediate lesson of Zeno's paradoxes. Eric Engle's comment is here relevant: "Zeno effectively asks 'How can motion be possible?' This paradox is arguably of little heuristic value today because we have since Einstein at least recognized that time and matter-energy are convertible elements, the same thing in fact. Thus rather than seeing a solid object, an arrow, existing at definite points in its trajectory, the correct view is to see a wave of energy following the arrow's trajectory with much greater mass/energy presence at certain instances of space time. That understanding is radically different from the ancients such as Zeno. For the ancients [Engle means some of the post-Parmenideans] , just as geometric points had no dimension -- just location, so also material loci were either void (kenon) or contained atoms. It is fair to say that geometric points and atoms corresponded to each other in the ancient conception of physics [this is actually true of the common sense of both the beginning functional and the beginning structural perspective, i.e. of common sense in general before the enlightened state of mind]. On this point we should note the structural similarity between Euclid's elements and Newton's principia. [Of course because both are formulations of common sense before the enlightened state or the mature stage of a perspective.] For some ancients (probably most in fact [i.e. for the common sense...]) matter and energy were not transmutable: rather the indestructible nature of atoms was a presumption of at least some ancients. Given these different assumptions about the nature of time and matter it is unsurprising that the heuristic value of Zeno's paradox is multi-variate and historically conditioned. In a world that (generally) presumed that matter and energy were very different quanta, Zeno's paradox forced one to ask what is meant by motion and time and to consider the presumptions underlying atomic theory. For the moderns [i.e. for the maturing mind of the structural perspective approaching the enlightened state], where both space/time and matter/energy are transmutable, the paradox really only illustrates the erroneous assumptions of pre-relativistic physics. Zeno's paradox collapses when one understands that matter/energy space/time are both elements of a unified field." (Ibid.) But this is precisely the point: the paradoxes are supposed to point out the fallaciousness of the common sense view of matter, space, and time, the transcendence of which, if not in the direction of Parmenides' metaphysics as Zeno intended it, should point the way to relativistic and quantum physics. Moreover, the lesson for us is that the paradoxes demonstrate the preeminence of logic over empirical experimentation. We have today arrived at quantum-relativistic world view entirely through experimentation. As said, if we have followed logic (e.g. Zeno's paradoxes), we would have arrived at such world view much earlier. Because we have trusted experimentation more than logic -- and since technology had not allowed accurate experimentation during the time of classical mechanics -- we could only have transcended the naive world view of the classical mechanics at the 20th century when the experimentation had at last achieved enough accuracy. As long as logic is well understood, if experimentation -- such as when someone demonstrates the fallacy of Zeno's paradoxes by walking across a distance -- disproves logic, then experimentation has to be wrong: for experimentation is always likely to be as yet inaccurate and logic, on the other hand, is eternally correct. This is extremely important to keep in mind since even today one frequently finds some saying "How logical double-talk dampened the empirical spirit of Greek physics" (e.g. Robert Logan, *The Alphabet Effect*, p. 114): "The arguments of Zeno and Parmenides would be dismissed today as logical double-talk or sophistry." (p. 115) Such is of course not true, and, together with the frequent assumption, among the modern "solutions", of Zeno's stupidity in his inability to understand the sometimes finite nature of sums of infinite or even to match the mind of a child, reflects simply the stupidity and arrogance of the modern who could not understand the ancient genius and is not intelligent and humble enough to consider the possibility that he may have not understood something. This is only the first point, for while both Zeno and Parmenides rank among the most misunderstood thinkers of all time, the former misunderstood the latter also: and this ties in with the structure of the evolution of consciousness itself.

Footnotes:

1. Peter Lynds, in "Zeno's Paradoxes: A Timely Solution" and also in "Time and Classical and Quantum Mechanics: Indeterminacy vs. Discontinuity", has advanced a view of solutions similar to (but not exactly the same as) ours (with regard to the paradox of the arrow), but since he excludes the last most important one of the stadium, does not formulate them strictly in terms of the uncertainty principle, and is confused about his solutions at times, we for the most part proceed in the following without references to his in order to spare confusion. His differs from ours in that he is denying the existence of "a precise static instant in time underlying a dynamical physical process", i.e. that Δt may be 0; while we formulate the problem below as the impossibility of a precise positioning during motion. On the right occasion we shall cite his to indicate temporal precedence. It should be said that his article can be considered a preliminary version of the present, although his came to my attention after I have already formulated my view and could not totally adjust mine to his or his to mine. Eric Engle has written a response to Lynds' (Zeno's Paradox: A Response to Mr. Lynds), of which citations will be given only at right occasions, since his confusing debunking of Lynds' confusing solutions will add further confusions if considered in full. The thick sediments of misunderstandings and confusions surrounding Zeno are indicative of the structure of the evolution of consciousness, to be considered in the next chapter.

2. I.e. The infinite series $\sum_{n=1}^{\infty}\left(\frac{1}{2}\right)^n$ = 1/2 + 1/4 + 1/8 + 1/16 + ... converges to 1 (this series is a special case of the geometric series). (From "Real Analysis" by Bert G. Wachsmuth.)

3. Here Lynds' view comes close to the same. "In the case of the Arrow paradox, there isn't an instant in time underlying the arrows [sic] motion at which it's [sic] volume would occupy just 'one block of space', and as its position is *constantly* changing in respect to time as a result, the arrow is never static and motionless." ("(c). The solution over 2500 years later" in "Zeno's Paradoxes: A Timely Solution".) The slight difference is that Lynds sees the uncertainty of position as the function of the non-instantaneity of time.

4. The Planck's length (L_p) is actually derived like this, with c, G, and h-cross (following Patricia Schwarz at superstring theory.com):

$$L_P = \sqrt{\frac{\hbar G_N}{c^3}} = 1.6 \times 10^{-33}\, cm$$

5. A most senseless misunderstanding of Zeno can be avoided if only some common sense be used. For example, this is how Nick Hugget in "Zeno's Paradoxes" (Stanford Encyclopedia of Philosophy) presents this last and most important paradox:

"The final paradox of motion runs as follows: picture three sets of three touching cubes -- all nine exactly the same, with side L m -- in relative motion. One set -- the *A*s --are at rest, and the others -- the *B*s and *C*s -- move from the left and right respectively, at a constant equal speed, *S m/s*. And suppose that at some moment the rightmost *B* is perfectly aligned with the middle *A*, and the leftmost *C* with the rightmost *A*: thus the edges of the rightmost *B* and leftmost *C* are exactly lined up. That is they are arranged as shown.

A A A
B B B
C C C

Now consider the later time at which the rightmost *A* and *B* are aligned; since the speeds of the *Bs* and *C*s are equal, at this moment the middle *A* will be aligned with the leftmost *C*. That is consider the moment when the blocks are configured thus.

A A A
B B B
C C C

This motion requires the rightmost *B* to move one block -- a distance *L m* -- to the right, at a speed of *S m/s*, so it takes *L/S s*. And the same motion also requires the leftmost *C* to move from just to the right of the rightmost *B* into alignment with the middle *B*, a distance a distance of 2*L m*. So far so good, but now Zeno concludes that since the *C*s are moving at *S m/s*, the motion must also take 2*L/S s*. And hence 'half the time [*L/S*] is equal to its double [2*L/S*]', since one and the same motion seems to take both times.

The unanimous verdict on Zeno is that he was hopelessly confused about relative velocity in this paradox. If the *B*s are moving with speed S m/s to the right with respect to the *As* and if the *Cs* are moving with speed *S m/s* to the left with respect to the *A*s then the *C*s are moving with speed *S*+*S* = 2*S m/s* to the left with respect to the *B*s. And so, as expected it takes the *C*s 2*L*/2*S* = *L/S s* to complete the motion after all.

This resolution notwithstanding, recent philosophers have attempted to put a new spin on Zeno's argument (it's arguable whether Zeno himself had anything like what follows in mind). This argument opposed the view that space and time are 'quantized', composed of smallest finite parts." Since even children are able to understand that, in everyday experience, the velocity of the other moving cart as experienced by a person on a moving cart going toward this other is V = V1 + V2, there is no way that Zeno would be so "hopelessly confused about relative velocity". What he had in mind must be some other scenario (which Aristotle did not transcribe it accurately due to misunderstanding), i.e. the attempted "new spin".

ACADEMY | previous section | Table of Content | next section | GALLERY